

CHAPTER 1

DPP 1.1

Subjective Type

1. The value of any given physical quantity may vary over a wide range, therefore, different units of same physical quantity are required. For example, the length of a pen can be easily measured in cm, the height of a tree can be measured in metres, the distance between two cities can be measured in kilometres and distance between two heavenly bodies can be measured in light year.

2. (a) We have the expression $v^2 = 2C_1x$. The unit in left side and right side of any expression should be same.

$$\text{Unit of left side expression} = \left[\frac{m}{s} \right]^2 = m^2/s^2$$

Hence unit of right side expression should also be same.

$$C_1 \times m = \frac{m^2}{s^2}; \text{ hence unit of } C_1 = m^2/s^2$$

- (b) We have the expression $x = C_1 \cos C_2t$

Unit of left side expression = m

On right side of expression $\cos C_2t$ is unitless. Hence unit of C_1 should also be m . The expression ' C_2t ' should be unitless hence unit of C_2 should be 'per sec' or '/sec'.

- (c) We have the expression $v = C_1e^{-C_2t}$

The term e^{-C_2t} is unitless. The power term C_2t should also be unitless, hence the unit of C_2 should be 'per sec' or '/sec'. The unit of C_1 should be same as unit on left side of expression. Hence the unit of C_1 should be 'm/s'.

Single Correct Answer Type

3. (d) Watt = Joule/second = Ampere $\times$ volt = Ampere² $\times$ Ohm
 4. (c) Impulse = change in momentum = $F \times t$
 So the unit of momentum will be equal to Newton-sec.
 5. (c) $F = \frac{Gm_1m_2}{d^2}$; $\therefore G = \frac{Fd^2}{m_1m_2} = Nm^2/kg^2$
 6. (c) Angular acceleration = $\frac{\text{Angular velocity}}{\text{Time}} = \frac{\text{rad}}{\text{sec}^2}$
 7. (b) Kg-m/sec is the unit of linear momentum
 8. (d) Because temperature is a fundamental quantity.
 9. (a) Energy (E) = $F \times d \Rightarrow F = \frac{E}{d}$, so Erg/metre can be the unit of force.
 10. (b) Potential energy = $mgh = g \left(\frac{cm}{\text{sec}^2} \right) cm = g \left(\frac{cm}{\text{sec}^2} \right)$
 11. (b) $\frac{\text{watt}}{\text{ampere}} = \text{volt}$

12. (c) Energy = force $\times$ distance, so if both are increased by 4 times, then energy will increase by 16 times.
 13. (a) Charge = current $\times$ time

Multiple Correct Answers Type

14. (a, b)

$$\text{We know that pressure} = \frac{\text{Force}}{\text{Area}}$$

$$\text{Pressure} = \frac{\text{Force} \times \text{Distance}}{\text{Area} \times \text{Distance}} = \frac{\text{Work}}{\text{Volume}} = \frac{\text{Energy}}{\text{Volume}}$$

DPP 1.2

Subjective Type

1. The given expression is $\frac{v}{t} = \frac{KAH}{L}$

$$\text{We can write } K = \frac{VL}{AHt}$$

The unit of L & H is same, hence the unit of K should be same as the unit of term $\frac{V}{At}$.

$$\text{We have } \frac{V}{At} \rightarrow \frac{m^s}{m^2 \text{sec}} \rightarrow \frac{m}{s}$$

Hence the unit of K is 'm/s'.

Single Correct Answer Type

2. (a) Physical quantity (p) = Numerical value (n) $\times$ Unit (u)
 If physical quantity remains constant then
 $n \propto 1/u \therefore n_1u_1 = n_2u_2$
 3. (c) $Y = \frac{F}{A} \cdot \frac{L}{\Delta L} = \frac{\text{dyne}}{\text{cm}^2} = \frac{10^{-5} \text{N}}{10^{-4} \text{m}^2} = 0.1 \text{ N/m}^2$
 4. (a) $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\text{Force/Area}}{\text{Dimensionless}} \Rightarrow Y = \text{Pressure}$
 5. (c) 1 fermi = 10^{-15} metre
 6. (d) Watt is a unit of power
 7. (c) $n_2 = n_1 \left(\frac{M_1}{M_2} \right)^1 \left(\frac{L_1}{L_2} \right)^1 \left(\frac{T}{T_2} \right)^{-2}$
 $= 100 \left(\frac{\text{gm}}{\text{kg}} \right)^1 \left(\frac{\text{cm}}{\text{m}} \right)^1 \left(\frac{\text{sec}}{\text{min}} \right)^{-2}$
 $= 100 \left(\frac{\text{gm}}{10^3 \text{gm}} \right)^1 \left(\frac{\text{cm}}{10^2 \text{cm}} \right)^1 \left(\frac{\text{sec}}{60 \text{sec}} \right)^{-2}$
 $n_2 = \frac{3600}{10^3} = 3.6$

8. (d) $P = nu \therefore n \propto \frac{1}{u}$

9. (c) $[x] = [br^2] \Rightarrow [b] = [x/r^2] = \text{km/s}^2$

10. (d) cr^2 must have dimensions of L
 $\Rightarrow c$ must have dimensions of L/T^2 i.e. LT^{-2} .

11. (b) Units of a and PV^2 are same and equal to $\text{dyne} \times \text{cm}^4$.

12. (b) $\text{Watt} = \frac{J}{\text{sec}} = \frac{\text{N-m}}{\text{sec}} = \text{kg} \frac{\text{m}^2}{\text{sec}^3}$

13. (b) $F = \frac{\text{Newton}}{\text{m}^2}$

$$\frac{hc}{d^2} = \text{joule-sec.} \left(\frac{\text{metre}}{\text{sec.}} \right) \frac{1}{(\text{metre})^2}$$

$$= \frac{\text{joule}}{\text{metre}} = \text{Newton}$$

$$\frac{hc}{d^4} = \left(\frac{hc}{d^2} \right) \frac{1}{d^2} = \frac{\text{Newton}}{\text{m}^2}. \text{ Hence } F = hc/d^4$$

$$\frac{hd}{c} = \frac{\text{Joule.sec}(\text{metre})^2}{\left(\frac{\text{metre}}{\text{sec.}} \right)} = \text{Newton-metre}^2 \text{ sec}^2$$

$$\frac{d^4}{hc} = \frac{1}{\left(\frac{hc}{d^4} \right)} = \frac{\text{m}^2}{\text{Newton}}$$

Matching Column Type

14. (a) (I) $1N = \frac{\text{kg m}}{\text{sec}^2} = \frac{\left(\frac{1 \text{ unit mass}}{\alpha} \right) \left(\frac{1 \text{ unit length}}{\beta} \right)}{\left(\frac{1 \text{ unit time}}{\gamma} \right)^2} = \frac{\gamma^2}{\alpha\beta}$

unit force in new system.

(II) $1J = \frac{1 \text{ kg}(1 \text{ m})^2}{(1 \text{ sec})^2} = \frac{\frac{1}{\alpha} \left(\frac{1}{\beta} \right)^2}{\left(\frac{1}{\gamma} \right)^2} \text{ unit energy}$

$$= \alpha^{-1} \beta^{-2} \gamma^2$$

(III) $1 \text{ pascal} = \frac{1 \text{ kg}}{(1 \text{ m})(1 \text{ sec})^2} = \frac{\frac{1}{\alpha}}{\frac{1}{\beta} \left(\frac{1}{\gamma} \right)^2} \text{ unit pressure}$

$$= \alpha^{-1} \beta \gamma^2$$

(IV) $\alpha_{ACME} = \alpha = \frac{(1 \text{ unit mass})(1 \text{ unit length})^2}{(1 \text{ unit time})^3}$

$$= \alpha \frac{\text{kg} \beta \text{m}}{(\gamma \text{ sec})^3} = \alpha^2 \beta^2 \gamma^3 \text{ watt}$$

15. (c) (I) Given $I = \frac{GM_e}{R_e^2}$

The unit of G can be found from newton gravitational law

$$F = \frac{GM_1 M_2}{r^2} \Rightarrow G = \frac{Fr^2}{M_1 M_2}$$

Hence unit of G is $\frac{\text{Nm}^2}{\text{kg}^2}$

Hence unit of $I = \left(\frac{\text{Nm}^2}{\text{kg}^2} \right) \cdot \text{kg} = \text{newton/kg}$

(II) Given $E = \frac{F}{Q}$

Hence the unit of E should be newton/coulomb.

(III) Given $R = \frac{V}{I}$

The SI unit of V is volt and that of I is ampere. Hence unit of R should be volt/ampere, this is called ohm (Ω).

(IV) Given $R = \frac{pV}{nT} = \frac{(\text{N/m}^2) \times \text{m}^3}{\text{mol} \times \text{kelvin}}$

$$= \frac{\text{Nm}}{\text{mol-kelvin}} = \frac{\text{joule}}{\text{mol-kelvin}}$$

DPP 1.3

Subjective Type

1. The dimensions of left hand side expression should be same as right hand side expression.

$$\left[\frac{\alpha}{\beta + \sqrt{d}} \right] = [F] = [MLT^{-2}]$$

The dimensional formula for β should be same as the dimensional formula for $\sqrt{d}$.

$$[\beta] = [\sqrt{d}] = [ML^{-3}]^{1/2} = [M^{1/2} L^{-3/2}]$$

$$\text{Hence } [\alpha] = [\beta][F] = [M^{1/2} L^{-3/2}] = [MLT^{-2}]$$

$$\Rightarrow [\alpha] = [M^{3/2} L^{-1/2} T^{-2}]$$

Single Correct Answer Type

2. (a) Pressure = $\frac{\text{Force}}{\text{Area}} = ML^{-1}T^{-2}$

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}} = ML^{-1}T^{-2}$$

3. (c) Strain = $\frac{\Delta L}{L} \Rightarrow$ dimensionless quantity

4. (c) Impulse = change in momentum so dimensions of both quantities will be same and equal to MLT^{-1}

5. (a) Momentum = $mv = [MLT^{-1}]$

$$\text{Impulse} = \text{Force} \times \text{Time} = [MLT^{-2}] \times [T] = [MLT^{-1}]$$

6. (b) Pressure = $\frac{\text{Force}}{\text{Area}} = \frac{\text{Energy}}{\text{Volume}} = ML^{-1}T^{-2}$

7. (a) By principle of dimensional homogeneity $\left[\frac{a}{V^2} \right] = [P]$

$$\therefore [a] = [P][V^2] = [ML^{-1}T^{-2}] \times [L^6] = [ML^5T^{-2}]$$

8. (d) By putting the dimensions of each quantity both the sides we get $[T^{-1}] = [M]^x [MT^{-2}]^y$

Now comparing the dimensions of quantities in both sides we

$$\text{get } x + y = 0 \text{ and } 2y = 1 \therefore x = -\frac{1}{2}, y = \frac{1}{2}$$

9. (c) $m = \text{linear density} = \text{mass per unit length} = \left[\frac{M}{L} \right]$

$A = \text{force} = [MLT^{-2}] \therefore [B] = \frac{[A]}{[m]} = \frac{[MLT^{-2}]}{[ML^{-1}]} = [L^2T^{-2}]$

This is same dimension as that of latent heat.

10. (b) $v \propto g^p h^q$ (given)

By substituting the dimension of each quantity and comparing the powers in both sides we get $[LT^{-1}] = [LT^{-2}]^p [L]^q$

$\Rightarrow p + q = 1, -2p = -1, \therefore p = \frac{1}{2}, q = \frac{1}{2}$

11. (d) $[\text{Planck constant}] = [ML^2T^{-1}]$ and $[\text{Energy}] = [ML^2T^{-2}]$

12. (a) By substituting dimension of each quantity in R.H.S. of option

(a) we get $\left[\frac{mg}{\eta r} \right] = \left[\frac{M \times LT^{-2}}{ML^{-1}T^{-1} \times L} \right] = [LT^{-1}]$.

This option gives the dimension of velocity.

13. (a) According to problem muscle $\times$ speed = power

$\therefore \text{Muscle} = \frac{\text{power}}{\text{speed}} = \frac{ML^2T^{-3}}{LT^{-1}} = MLT^{-2}$

14. (b) By substituting the dimension of given quantities $[ML^{-1}T^{-2}]^x [MT^{-2}]^y [LT^{-1}]^z = [MLT]^0$

By comparing the power of M, L, T in both sides

$x + y = 0$ (i)

$-x + z = 0$ (ii)

$-2x - 2y - z = 0$ (iii)

The only values of x, y, z satisfying (i), (ii) and (iii) corresponds to (b).

Fill in the Blanks Type

15. $Q = mL$

$\Rightarrow L = \frac{Q}{m}$ (Heat is a form of energy)

$= \frac{ML^2T^{-2}}{M} = [M^0L^2T^{-2}]$

16. $F = \frac{Gm_1m_2}{d^2} \Rightarrow G = \frac{Fd^2}{m_1m_2}$

$\therefore [G] = \frac{[MLT^{-2}][L^2]}{[M^2]} = [M^{-1}L^3T^{-2}]$

17. Angular velocity = $\frac{\theta}{t}$, $[\omega] = \frac{[M^0L^0T^0]}{[T]} = [T^{-1}]$

18. Power = $\frac{\text{Work done}}{\text{Time}} = \left[\frac{ML^2T^{-2}}{T} \right] = [ML^2T^{-3}]$

19. Couple = Force $\times$ Arm length = $[MLT^{-2}][L] = [ML^2T^{-2}]$

20. Impulse = Force $\times$ Time = $[MLT^{-2}][T] = [MLT^{-1}]$

21. $E = h\nu \Rightarrow [ML^2T^{-2}] = [h][T^{-1}] \Rightarrow [h] = [ML^2T^{-1}]$

22. Calorie is the unit of heat, i.e., energy.

So dimensions of energy = ML^2T^{-2}

Subjective Type

1. (i) $[a] = [Ux^2] = ML^2T^{-2}L^2 = ML^4T^{-2}$
 $[b] = [Ux] = ML^2T^{-2}L = ML^3T^{-2}$

(ii) $T = 4\pi a \sqrt{\frac{ma}{b^2}} = 4\pi \sqrt{\frac{ma^3}{b^2}}$

Dimension of RHS = $\left[4\pi \sqrt{\frac{ma^3}{b^2}} \right] = \sqrt{\frac{MM^3L^{12}T^{-6}}{M^2L^6T^{-4}}} T$

So, his answer is dimensionally incorrect.

Single Correct Answer Type

2. (b) $L \propto v^x A^y F^z \Rightarrow L = kv^x A^y F^z$

Putting the dimensions in the above relation

$[ML^2T^{-1}] = k[LT^{-1}]^x [LT^{-2}]^y [MLT^{-2}]^z$

$\Rightarrow [ML^2T^{-1}] = k[M^xL^{x+y+z}T^{-x-2y-2z}]$

Comparing the powers of M, L and T

$z = 1$ (i)

$x + y + z = 2$ (ii)

$-x - 2y - 2z = -1$ (iii)

On solving (i), (ii) and (iii) $x = 3, y = -2, z = 1$

So dimension of L in terms of v, A and F

$[L] = [Fv^3A^{-2}]$

3. (a) Quantities having different dimensions can only be divided or multiplied but they cannot be added or subtracted.

4. (b) From the principle of dimensional homogeneity $[a] = \left[\frac{F}{t} \right] = [MLT^{-3}]$ and $[b] = \left[\frac{F}{t^2} \right] = [MLT^{-4}]$

5. (b) Let $[G] \propto c^x g^y p^z$

by substituting the following dimensions:

$[G] = [M^{-1}L^3T^{-2}], [c] = [LT^{-1}], [g] = [LT^{-2}]$

$[p] = [ML^{-1}T^{-2}]$

and by comparing the powers of both sides

we can get $x = 0, y = 2, z = -1$

$\therefore [G] \propto c^0 g^2 p^{-1}$

6. (a) Let $T \propto S^x r^y \rho^z$

by substituting the dimension of $[T] = [T]$

$[S] = [MT^{-2}], [r] = [L], [\rho] = [ML^{-3}]$

and by comparing the power of both the sides

$x = -1/2, y = 3/2, z = 1/2$

so $T \propto \sqrt{\rho r^3/S} \Rightarrow T = k \sqrt{\frac{\rho r^3}{S}}$

DPP 1.5

Subjective Type

7. (a) Let $F \propto P^x V^y T^z$

by substituting the following dimensions:

$$[P] = [ML^{-1}T^{-2}] \quad [V] = [LT^{-1}], \quad [T] = [T]$$

and comparing the dimension of both sides

$$x = 1, y = 2, z = 2, \text{ so } F = PV^2T^2$$

8. (b) Let $m \propto E^x V^y F^z$

By substituting the following dimensions:

$$[E] = [ML^2T^{-2}], \quad [V] = [LT^{-1}], \quad [F] = [MLT^{-2}]$$

and by equating the both sides

$$x = 1, y = -2, z = 0. \text{ So } [m] = [E V^{-2}]$$

9. (d) $x = Ay + B \tan Cz$

From the dimensional homogeneity

$$[x] = [Ay] = [B] \Rightarrow \left[\frac{x}{A} \right] = [y] = \left[\frac{B}{A} \right]$$

$$[Cz] = [M^0 L^0 T^0] = \text{Dimensionless}$$

x and B ; C and Z^{-1} ; y and $\frac{B}{A}$ have the same dimension but x

and A have the different dimensions.

10. (c) Let $m \propto C^x G^y h^z$

By substituting the following dimensions:

$$[C] = [LT^{-1}], \quad [G] = [M^{-1}L^3T^{-2}] \text{ and } [h] = [ML^2T^{-1}]$$

Now comparing both sides we will get

$$x = 1/2; y = -1/2, z = +1/2$$

$$\text{So } m \propto C^{1/2} G^{-1/2} h^{1/2}$$

11. (a) Let $m = KF^a L^b T^c$

Substituting the dimension of

$$[F] = [MLT^{-2}], \quad [C] = [L] \text{ and } [T] = [T]$$

and comparing both sides, we get $m = FL^{-1}T^{-2}$

12. (b) As $x = Ka^m \times t^n$

$$[M^0 L^0] = [LT^{-2}]^m [T]^n = [L^m T^{-2m+n}]$$

$$\therefore m = 1 \text{ and } -2m + n = 0 \Rightarrow n = 2.$$

13. (b) $E = KF^a A^b T^c$

$$[ML^2T^{-2}] = [MLT^{-2}]^a [LT^{-2}]^b [T]^c$$

$$[ML^2T^{-2}] = [M^a L^{a+b} T^{-2a-2b+c}]$$

$$\therefore a = 1, a + b = 2 \Rightarrow b = 1$$

$$\text{and } -2a - 2b + c = -2 \Rightarrow c = 2$$

$$\therefore E = KFAT^2.$$

14. (d) CGS SI

$$N_1 U_1 = N_2 U_2$$

$$\therefore N_1 [M_1 L_1^{-3}] = N_2 [M_2 L_2^{-3}]$$

$$N_2 = N_1 \left[\frac{M_1}{M_2} \right] \times \left[\frac{L_1}{L_2} \right]^{-3} = 0.625 \left[\frac{1g}{1kg} \right] \times \left[\frac{1cm}{1m} \right]^{-3}$$

$$= 0.625 \times 10^{-3} \times 10^6 = 625$$

15. (a) Plane angle is dimensionless

$$[WA] = [v]$$

$$[W]L = LT^{-1}$$

$$[W] = T^{-1}$$

$$\left[\frac{k}{\sqrt{m}} t \right] = 1$$

$$k = \frac{m}{t^2} = MT^{-2}$$

$$\left(\sqrt{\frac{k}{m}} \right) \{t\} = 1 \Rightarrow \sqrt{\frac{k}{m}} = T^{-1}$$

1. Given, $t_1 = 39.6$ s, $t_2 = 39.9$ s and $t_3 = 39.5$ s

Least count of measuring instrument = 0.1 s

(As measurements have only one decimal place) Precision in the

measurement = Least count of the measuring instrument = 0.1 s

Mean value of time for 20 oscillations is given by

$$t = \frac{t_1 + t_2 + t_3}{3}$$

$$= \frac{39.6 + 39.9 + 39.5}{3} = 39.7 \text{ s}$$

Absolute errors in the measurements

$$\Delta t_1 = t - t_1 = 39.7 - 39.6 = 0.1 \text{ s}$$

$$\Delta t_2 = t - t_2 = 39.7 - 39.9 = -0.2 \text{ s}$$

$$\Delta t_3 = t - t_3 = 39.7 - 39.5 = 0.2 \text{ s}$$

$$\text{Mean absolute error} = \frac{|\Delta t_1| + |\Delta t_2| + |\Delta t_3|}{3}$$

$$= \frac{0.1 + 0.2 + 0.2}{3} = \frac{0.5}{3} = 0.17 \approx 0.2$$

(rounding off upto one decimal place)

$$\therefore \text{Accuracy of measurement} = \pm 0.2 \text{ s}$$

2. Given, physical quantity is $X = a^2 b^3 c^{5/2} d^{-2}$

Maximum percentage error in X is

$$\frac{\Delta X}{X} \times 100 = \pm \left[2 \left(\frac{\Delta a}{a} \times 100 \right) + 3 \left(\frac{\Delta b}{b} \times 100 \right) \right.$$

$$\left. + \frac{5}{2} \left(\frac{\Delta c}{c} \times 100 \right) + 2 \left(\frac{\Delta d}{d} \times 100 \right) \right]$$

$$= \pm \left[2(1) + 3(2) + \frac{5}{2}(3) + 2(4) \right] \%$$

$$= \pm \left[2 + 6 + \frac{15}{2} + 8 \right] = \pm 23.5\%$$

$$\therefore \text{Percentage error in quantity } X = \pm 23.5\%$$

Mean absolute error in $X = \pm 0.235 = \pm 0.24$ (rounding-off up to two significant digits)

The calculated value of x should be round-off up to two significant digits.

$$\therefore X = 2.8$$

Single Correct Answer Type

3. (b) Number of significant figures are 3, because 10^3 is decimal multiplier.

4. (b) If the number is less than 1, the zero (s) on the right of decimal point and before the first non-zero digit are not significant. In 0.06900, the underlined zeroes are not significant. Hence, number of significant figures are four (6900).

5. (b) The sum of the numbers can be calculated as 663.821 arithmetically. The number with least decimal places is 227.2 is correct to only one decimal place. The final result should, therefore be rounded off to one decimal place, i.e., 664.

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6. (c) In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures. In this question, density should be reported to two significant figures.

$$\text{Density} = \frac{4.237 \text{ g}}{2.5 \text{ cm}^3} = 1.6948$$

As rounding off the number, we get density = 1.7

7. (d) Rounding off 2.745 to 3 significant figures it would be 2.74.
Rounding off 2.735 to 3 significant figures it would be 2.74.

8. (a) If Δx is error in a physical quantity, then relative error is calculated as $\frac{\Delta x}{x}$. Given, length $l = (16.2 \pm 0.10) \text{ cm}$

$$\text{Breadth } b = (10.1 \pm 0.1) \text{ cm}$$

$$\text{Area } A = l \times b$$

$$= (162 \text{ cm}) \times (10.1 \text{ cm}) = 163.62 \text{ cm}^2$$

Rounding off to three significant digits, area $A = 164 \text{ cm}^2$

$$\begin{aligned} \frac{\Delta A}{A} &= \frac{\Delta l}{l} + \frac{\Delta b}{b} = \frac{0.1}{16.2} + \frac{0.1}{10.1} \\ &= \frac{1.01 + 1.62}{16.2 \times 10.1} = \frac{2.63}{163.62} \end{aligned}$$

$$\begin{aligned} \Rightarrow \Delta A &= A \times \frac{2.63}{163.62} \\ &= 163.62 \times \frac{2.63}{163.62} = 2.63 \text{ cm}^2 \end{aligned}$$

$\Delta A = 3 \text{ cm}^2$ (By rounding off to one significant figure)

$$\text{Area, } A = A \pm \Delta A = (164 \pm 3) \text{ cm}^2$$

9. (a) $\frac{1}{20} = 0.05$

$\therefore$ Decimal equivalent up to 3 significant figures is 0.0500

10. (c) Since for 50.14 cm, significant number = 4 and for 0.00025, significant numbers = 2

11. (c) Given, $L = 2.331 \text{ cm}$
 $= 2.33$ (correct up to two decimal places)

$$\text{and } B = 2.1 \text{ cm} = 2.10 \text{ cm}$$

$$\therefore L + B = 2.33 + 2.10 = 4.43 \text{ cm} = 4.4 \text{ cm}$$

Since minimum significant figure is 2.

12. (a) $A = 2.5 \text{ ms}^{-1} \pm 0.5 \text{ ms}^{-1}$, $B = 0.10 \text{ s} \pm 0.01 \text{ s}$

$$x = AB = (2.5)(0.10) = 0.25 \text{ m}$$

$$\begin{aligned} \frac{\Delta x}{x} &= \frac{\Delta A}{A} + \frac{\Delta B}{B} \\ &= \frac{0.5}{2.5} + \frac{0.01}{0.10} = \frac{0.05 + 0.025}{0.25} = \frac{0.075}{0.25} \end{aligned}$$

$$\Delta x = 0.075 = 0.08 \text{ m}$$

Rounding off to two significant figures.

$$AB = (0.25 \pm 0.08) \text{ m}$$

13. (d) Given, $A = 1.0 \text{ m} \pm 0.2 \text{ m}$, $B = 2.0 \text{ m} \pm 0.2 \text{ m}$

$$\text{Let, } Y = \sqrt{AB} = \sqrt{(1.0)(2.0)} = 1.414 \text{ m}$$

Rounding off to two significant digit $Y = 1.4 \text{ m}$

$$\begin{aligned} \frac{\Delta Y}{Y} &= \frac{1}{2} \left[\frac{\Delta A}{A} + \frac{\Delta B}{B} \right] \\ &= \frac{1}{2} \left[\frac{0.2}{1.0} + \frac{0.2}{2.0} \right] = \frac{0.6}{2 \times 2.0} \end{aligned}$$

$$\Rightarrow \Delta Y = \frac{0.6Y}{2 \times 2.0} = \frac{0.6 \times 1.4}{2 \times 2.0} = 0.212$$

Rounding off to one significant digit

$$\Delta Y = 0.2 \text{ m}$$

Thus, correct value for

$$\sqrt{AB} = r + \Delta r = 1.4 \pm 0.2 \text{ m}$$

14. (a) All given measurements are correct up to two decimal places. As here 5.00 mm has the smallest unit and the error in 5.00 mm is least (commonly taken as 0.01 mm if not specified), hence, 5.00 mm is most precise.

15. (a) Given length $l = 5 \text{ cm}$

Now, checking the errors with each options one by one, we get

$$\Delta l_1 = 5 - 4.9 = 0.1 \text{ cm}$$

$$\Delta l_2 = 5 - 4.805 = 0.195 \text{ cm}$$

$$\Delta l_3 = 5.25 - 5 = 0.25 \text{ cm}$$

$$\Delta l_4 = 5.4 - 5 = 0.4 \text{ cm}$$

Error Δl_1 is least.

Hence 4.9 cm is most precise

DPP 1.6

Single Correct Answer Type

1. (c) $T = 2\pi\sqrt{l/g} \Rightarrow T^2 = 4\pi^2 l/g \Rightarrow g = \frac{4\pi^2 l}{T^2}$

$$\text{Here \% error in } l = \frac{1 \text{ mm}}{100 \text{ cm}} \times 100 = \frac{0.1}{100} \times 100 = 0.1\%$$

$$\text{and \% error in } T = \frac{0.1}{2 \times 100} \times 100 = 0.05\%$$

$$\therefore \% \text{ error in } g = \% \text{ error in } l + 2(\% \text{ error in } T) = 0.1 + 2 \times 0.05 = 0.2\%$$

2. (b) $\therefore E = \frac{1}{2}mv^2$

$\therefore$ % Error in K.E.

$$= \% \text{ error in mass} + 2 \times \% \text{ error in velocity}$$

$$= 2 + 2 \times 3 = 8\%$$

3. (b) $\therefore V = \frac{4}{3}\pi r^3$

$$\therefore \% \text{ error in volume} = 3 \times \% \text{ error in radius} = 3 \times 1 = 3\%$$

4. (c) Mean time period $T = 2.00 \text{ sec}$

$$\text{and Mean absolute error} = \Delta T = 0.05 \text{ sec}$$

To express maximum estimate of error, the time period should be written as $(2.00 \pm 0.05) \text{ sec}$

5. (b) Here, $S = (13.8 \pm 0.2) \text{ m}$

$$\text{and } t = (4.0 \pm 0.3) \text{ sec}$$

Expressing it in percentage error, we have,

$$S = 13.8 \pm \frac{0.2}{13.8} \times 100\% = 13.8 \pm 1.4\%$$

$$\text{and } t = 4.0 \pm \frac{0.3}{4} \times 100\% = 4 \pm 7.5\%$$

$$\therefore V = \frac{s}{t} = \frac{13.8 \pm 1.4}{4 \pm 7.5} = (3.45 \pm 0.3) \text{ m/s}$$

6. (c) % error in velocity = %error in L + %error in t

$$= \frac{0.2}{13.8} \times 100 + \frac{0.3}{4} \times 100$$

$$= 1.44 + 7.5 = 8.94\%$$

7. (d) $a = b^\alpha c^\beta d^\gamma e^\delta$

So maximum error in a is given by

$$\left(\frac{\Delta a}{a} \times 100 \right)_{\max} = \alpha \cdot \frac{\Delta b}{b} \times 100 + \beta \cdot \frac{\Delta c}{c} \times 100 + \gamma \cdot \frac{\Delta d}{d} \times 100 + \delta \cdot \frac{\Delta e}{e} \times 100$$

$$= (\alpha b_1 + \beta c_1 + \gamma d_1 + \delta e_1)\%$$

8. (a) Weight in air $(5.00 \pm 0.05)N$

Weight in water $= (4.00 \pm 0.05)N$

Loss of weight in water $= (1.00 \pm 0.1)N$

Now relative density = $\frac{\text{weight in air}}{\text{weight loss in water}}$

i.e. $R \cdot D = \frac{5.00 \pm 0.05}{1.00 \pm 0.1}$

Now relative density with max permissible error

$$= \frac{5.00}{1.00} \pm \left(\frac{0.05}{5.00} + \frac{0.1}{1.00} \right) \times 100 = 5.0 \pm (1 + 10)\%$$

$$= 5.0 \pm 11\%$$

9. (b) Average value = $\frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$

$$= 2.62 \text{ sec}$$

Now $|\Delta T_1| = 2.63 - 2.62 = 0.01$

$|\Delta T_2| = 2.62 - 2.56 = 0.06$

$|\Delta T_3| = 2.62 - 2.42 = 0.20$

$|\Delta T_4| = 2.71 - 2.62 = 0.09$

$|\Delta T_5| = 2.80 - 2.62 = 0.18$

Mean absolute error

$$\Delta T = \frac{|\Delta T_1| + |\Delta T_2| + |\Delta T_3| + |\Delta T_4| + |\Delta T_5|}{5}$$

$$= \frac{0.54}{5} = 0.108 = 0.11 \text{ sec}$$

10. (c) Volume of cylinder $V = \pi r^2 l$

Percentage error in volume

$$\frac{\Delta V}{V} \times 100 = \frac{2\Delta r}{r} \times 100 + \frac{\Delta l}{l} \times 100$$

$$= \left(2 \times \frac{0.01}{2.0} \times 100 + \frac{0.1}{5.0} \times 100 \right) = (1 + 2)\% = 3\%$$

11. (c) $Y = \frac{4MgL}{\pi D^2 l}$ so maximum permissible error in Y

$$= \frac{\Delta Y}{Y} \times 100 = \left(\frac{\Delta M}{M} + \frac{\Delta g}{g} + \frac{\Delta L}{L} + \frac{2\Delta D}{D} + \frac{\Delta l}{l} \right) \times 100$$

$$= \left(\frac{1}{300} + \frac{1}{981} + \frac{1}{2820} + 2 \times \frac{1}{41} + \frac{1}{87} \right) \times 100$$

$$= 0.065 \times 100 = 6.5\%$$

12. (d) Kinetic energy $E = \frac{1}{2}mv^2$

$$\therefore \frac{\Delta E}{E} \times 100 = \frac{v'^2 - v^2}{v^2} \times 100$$

$$= [(1.5)^2 - 1] \times 100$$

$$\therefore \frac{\Delta E}{E} \times 100 = 125\%$$

13. (d) Percentage error in A

$$= \left(2 \times 1 + 3 \times 3 + 1 \times 2 + \frac{1}{2} \times 2 \right) \% = 14\%$$

14. (d) $\therefore$ Density, $\rho = \frac{M}{V} = \frac{M}{\pi r^2 L}$

$$\Rightarrow \frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + 2 \frac{\Delta r}{r} + \frac{\Delta L}{L}$$

$$= \frac{0.003}{0.3} + 2 \times \frac{0.005}{0.5} + \frac{0.06}{6}$$

$$= 0.01 + 0.02 + 0.01 = 0.04$$

$$\therefore \text{Percentage error} = \frac{\Delta \rho}{\rho} \times 100 = 0.04 \times 100 = 4\%$$

CHAPTER 2

DPP 2.1

Subjective Type

1. Clearly $\sin(105^\circ) = \sin(60^\circ + 45^\circ)$

$$= \sin 60^\circ \cdot \cos 45^\circ + \cos 60^\circ \sin 45^\circ$$

Using $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{1}{4}(\sqrt{6} + \sqrt{2})$$

2. We know that $22\frac{1^\circ}{2} \times 2 = 45^\circ$

Taking $22\frac{1^\circ}{2} = A$; we get $2A = 45^\circ$

$\therefore$ Using, $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$, we have

$$\frac{2 \tan A}{1 - \tan^2 A} = 1 \quad [\because \tan 2A = \tan 45^\circ = 1]$$

$$\Rightarrow \tan^2 A + 2 \tan A - 1 = 0$$

$$\Rightarrow \tan A = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

Now, since $22\frac{1^\circ}{2}$ is acute angle, so $\tan 22\frac{1^\circ}{2} = +ve$

$$\therefore \tan 22\frac{1^\circ}{2} = \sqrt{2} - 1 \quad [\text{Omitting } -1 - \sqrt{2}]$$

3. $\therefore 2(\sin^2 x - \cos^2 x) = 1$

$$\Rightarrow 2[\sin^2 x - (1 - \sin^2 x)] = 1 \quad [\because \sin^2 x + \cos^2 x = 1]$$

$$\Rightarrow 4 \sin^2 x = 3 \Rightarrow \sin^2 x = \frac{3}{4} \Rightarrow \sin x = \pm \frac{\sqrt{3}}{2}$$

$$\Rightarrow x = 60^\circ, 120^\circ, 240^\circ, 300^\circ, \text{ etc.}$$

Alternatively : Using $\sin^2 x - \cos^2 x = -\cos^2 x$, we have $-\cos 2x$

$$= \frac{1}{2} \Rightarrow \cos 2x = -\frac{1}{2}$$

4. $\sqrt{26} = (25+1)^{1/2} = 5\left[1 + \frac{1}{25}\right]^{1/2}$

$$= 5\left[1 + \frac{1}{2} \cdot \frac{1}{25}\right]$$

$$= 5[1 + 0.02]$$

$$= 5[1.02] = 5.10$$

$$= 5.10 \text{ (correct to two places of decimal)}$$

5. Given $g \propto \frac{1}{r^2}$

If g be the value required, then

$$\frac{g}{9.81} = \left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{g}{9.81} = \left[\frac{R+h}{R}\right]^{-2}$$

$$\Rightarrow g = 9.81 \left[1 + \frac{h}{R}\right]^{-2} = 9.81 \left[1 - 2\frac{h}{R}\right]$$

$$= 9.81 \left[1 - 2\left(\frac{64}{6400}\right)\right] = 9.81 \left[1 - \left(\frac{2}{100}\right)\right]$$

$$= 9.81 \left[1 - \left(\frac{1}{50}\right)\right] = 9.81 \left[\frac{49}{50}\right] = 9.61 \text{ m/s}^2$$



Single Correct Answer Type

6. (a) $18 \times \frac{\pi}{180} = \frac{\pi}{10} \text{ rad}$

7. (a) $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

8. (c) $\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$

9. (b) $\sin(90^\circ + \theta) = \cos \theta$

10. (a) $\cos(30^\circ) = \frac{\sqrt{3}}{2}$

11. (c) $\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$

Multiple Correct Answers Type

12. (a, b, d)

$$\tan 45^\circ = 1$$

$$\sin 90^\circ = 1$$

$$\cos 0^\circ = 1$$

13. (a, b)

$$\sin^\circ = 0$$

$$\tan 0^\circ = 0$$

Matching Column Type

14. (a) $\sin 37^\circ = \frac{3}{5}$

(b) $\cos 127^\circ = \cos(180^\circ - 53^\circ) = -\cos 53^\circ = -\frac{3}{5}$

Since, $\cos(180^\circ - \theta) = -\cos \theta$

(c) $\tan 307^\circ = \tan(360^\circ - 53^\circ) = -\tan 53^\circ = -\frac{4}{3}$

$$(d) \cos 307^\circ = \cos(360^\circ - 53^\circ) = \cos 53^\circ = \frac{3}{5}$$

$$(e) \cos(-53^\circ) = \cos 53^\circ = \frac{3}{5}$$

DPP 2.2

Subjective Type

1. We have the relation $v = ut + at$. If we compare this relation with $y = mx + c = c + mx$, it is straight line relation

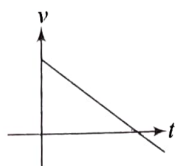
(i) We have $v = u + at$

Compare with $y = c + mx$

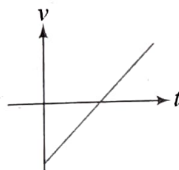
Have u is positive and a is negative.

Hence the straight line should have positive intercept and negative slope.

The graph should be



(ii) Here initial velocity is negative and acceleration is positive. The graph should have positive slope and negative intercept.



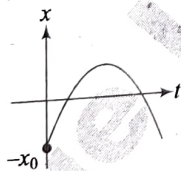
2. We have the relation $x = x_0 + ut + \frac{1}{2}at^2$

This relation is quadratic relation, it means the graph of x and t should be parabola. We have parabolic relation

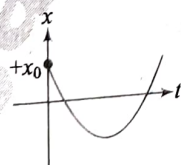
$$y = Ax^2 + Bx + C$$

$$\text{or } y = C + Bx + Ax^2$$

(i) Here x_0 and acceleration (a) both are negative. Hence the parabola should open down. Hence the graph should be like this



(ii) In this case initial position and acceleration both are positive hence the parabola should open up



Single Correct Answer Type

3. (c) Given $F = -10x$

The standard equation of straight line is $y = mx + c$

If we compare given equation with the standard equation of

straight line, the intercept is zero and slope is negative. Hence the graph is passing through origin.

Hence, option (c) is correct.

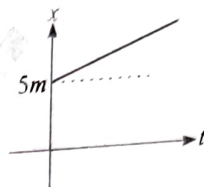
4. (a) $U = 5x^2$, it is the equation of parabola with concavity is upward. At $x = 0$, $U = 0$, it means graph is passing through origin. Hence, option (a) is correct.

5. (c) We know velocity $v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$

$$\Rightarrow 2 = \frac{x - 5}{t - 0}$$

$$\Rightarrow x = 2t + 5$$

If we compare with $y = mx + c$, the graph will be a straight line with positive slope with positive intercept



6. (c) During the fall, $v = 0 + gt$ i.e. $v = gt$

Which is of the form $y = mx$

a straight line passing through the origin.

The velocity is negative, as it is directed downwards. Next, when the stone rebounds, with a velocity v'

$$\text{Then } v = v' - gt$$

$$\text{or } v = -gt + v'$$

which is again a straight line with negative slope.

7. (c) $y = x^2 + 2x - 3$,

If we compare with $y = ax^2 + bx + c$

The coefficient of x^2 is positive hence parabola opens up

$$\text{At } x = 0, y = -3$$

$$\text{At } y = 0, x = \frac{-2 \pm \sqrt{4 + 12}}{2 \times 1}$$

$$= \frac{-2 \pm 4}{2} = -1 \pm 2 = -3, 1$$

Hence, option (c) is correct.

8. (b) The standard equation of straight line is

$$y = mx + c \quad \therefore v = mx + c$$

$$\text{at } x = 0, v = 10 \text{ m/s}$$

$$\therefore 10 = m \times 0 + c \text{ or } c = 10$$

$$\text{or } v = mx + 10$$

$$\text{At } x = 1, v = 0 \quad \therefore 0 = m \times 1 + 10$$

$$\text{or } m = -10 \quad \therefore v = -10x + 10$$

9. (d) Let ' a ' be the acceleration, then

$$\text{using } v^2 = u^2 + 2as$$

$$\text{We have } v^2 = 2a \left[s - \left(-\frac{u^2}{2a} \right) \right]$$

$$\text{Which is of the form } y^2 = 4A(x - h)$$

Clearly, it is a parabola, with vertex at $(h, 0)$ and axis, as x axis.

Matching Column Type

10. For graph A, C, and D the inclination of lines with positive x -direction is less than 90° , hence the slope will be positive. For graph B, the inclination of the line with positive x -direction is greater than 90° , hence its slope should be negative.

The line in graph A passes through origin, hence its intercept is zero. The lines in graph B & C cut positive side of y -axis hence their intercept should be positive.

The line of graph D, of y -axis hence its intercept should be negative.

DPP 2.3

Single Correct Answer Type

1. (b) $y = \sin x^2$

$$\frac{dy}{dx} = \cos(x^2) \frac{d}{dx}(x^2) = 2x \cos(x^2)$$

2. (b) $y = 2 \sin^2 \theta + \tan \theta$

$$\begin{aligned} \frac{dy}{d\theta} &= 2 \times 2 \tan \theta \cos \theta + \sec^2 \theta \\ &= 2 \sin 2\theta + \sec^2 \theta \end{aligned}$$

3. (a) $y = \sin x, x = 3t$

$$\frac{dy}{dx} = \cos x, \frac{dx}{dt} = 3$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = 3 \cos x = 3 \cos(3t)$$

4. (b) $y = \sin(t^2) \Rightarrow \frac{dy}{dt} = 2t \cos(t^2)$

$$\begin{aligned} \frac{d^2y}{dt^2} &= \cos t^2 \frac{d}{dt}(2t) + \frac{d}{dt} 2t \cos t^2 \\ &= 2 \cos t^2 + 2t(-\sin t^2) 2t \\ &= 2 \cos(t^2) - 4t^2 \sin(t^2) \end{aligned}$$

5. (a) $v = \frac{ds}{dt} = 10 - 0.2t$

For $v = 0$

$$t = \frac{10}{0.2} = 50 \text{ sec}$$

6. (a) $y = x^2 \sin x + \frac{3x}{\tan x}$

$$\begin{aligned} \frac{dy}{dx} &= x^2 \frac{d}{dx} \sin x + \sin x \frac{d}{dx}(x^2) \\ &\quad + \frac{\tan \frac{d}{dx}(3x) - 3x \frac{d}{dx}(\tan x)}{\tan^2 x} \\ \frac{dy}{dx} &= x^2 \cos x + 2x \sin x + \frac{3 \tan x - 3x \sec^2 x}{\tan^2 x} \end{aligned}$$

Comprehension Type

7. (b) $x = ut + \frac{1}{2}at^2$

$$dx = u(1) + \frac{1}{2}a(2t) = u + at$$

8. (a) $\frac{du}{dt} = 0 + a(1) = a$

9. (a) $q = 3 \sin 3t$

$$q = 2 \sin 3 \times \frac{\pi}{6} = 3 \text{ Coulombs}$$

10. (a) $i = \frac{dq}{dt} = 3 \times 3 \cos(3t)$

$$i \Big|_{t=\frac{\pi}{9}} = 9 \cos\left(3 \cdot \frac{\pi}{9}\right) = 9 \cos\left(\frac{\pi}{3}\right) = \frac{9}{2} \text{ A}$$

11. (b) $v = 2t^2 - 3t^2$

$$a = \frac{dv}{dt} = 6t^2 - 6t = 6t(t-1)$$

12. (a), 13. (b) $v = 2t^3 - 3t^2$

$$\frac{dv}{dt} = a = 6t(t-1)$$

$$\frac{dv}{dt} = 0$$

$$t = 0, 1 \text{ sec}$$

$$\frac{d^2v}{dt^2} = 12t - 6 \Rightarrow \left(\frac{d^2v}{dt^2}\right)_{t=0} = -6$$

$$\left(\frac{d^2v}{dt^2}\right)_{t=1} = 6$$

At $t = 0$, the sign of double derivative is negative, hence velocity will be maximum at 12:00 noon. At $t = 1$, the sign of double derivative is positive hence the speed will be minimum at 1:00 P.M.

14. (b) $v = 2t^3 - 3t^2 \Rightarrow t = 0 \Rightarrow v = 0$

Fill In The Blanks

15. $f(x) = y = 3x^2 + 2x + 4$

$$f(2) = 3(2)^2 + 2(2) + 4 = 20$$

$$f[f(2)] = 3(20)^2 + 2(20) + 4 = 1244$$

16. $\frac{d}{dx}(x^2) = 2x$

17. $y = x^{-4}$

$$\frac{dy}{dx} = -\frac{4}{x^5}$$

18. $y = 2x^2 + 3x$

$$\frac{dy}{dx} = 4x + 3$$

19. $x^2 \propto y^3$

$$x^2 = ky^3$$

when $x = 3$ and $y = 4$

$$9 = k \cdot 64 \Rightarrow k = \frac{9}{64}$$

$$\Rightarrow y^3 = \frac{x^2}{k} = \frac{1}{3 \times 9} \times 64 \Rightarrow y^3 = \frac{4^3}{3^3} \Rightarrow y = \frac{4}{3}$$

20. $y = 2 \sin x$

$$\frac{dy}{dx} = 2 \cos x$$

21. $y = \sin x$

$$\frac{dy}{dx} = \cos x$$

$$\frac{d^2y}{dx^2} = -\sin x$$

22. $\frac{dx}{dt} = v$, $\frac{d^2x}{dt^2} = \therefore$ acceleration

23. $y = x^3$

$$\frac{dy}{dx} = 3x^2 \Rightarrow \frac{d^2y}{dx^2} = 6x$$

24. $v = 2t^4$

$$a = \frac{dv}{dt} = 2 \times 4t^3 = 8t^3$$

25. Let $u = (\omega x + \phi)$

$$\left(\frac{dy}{dt}\right) = \frac{dy}{du} \times \frac{du}{dt}$$

$$= 2 \cos(\omega x + \phi) \times \omega = 2\omega \cos(\omega x + \phi)$$

26. $y = x^2 \sin x$

$$\frac{dy}{dx} = x^2 \frac{d}{dx} \sin x + \sin x \frac{d}{dx} (x^2)$$

$$= x^2 \cos x + 2x \sin x$$

27. $y = \tan x \cos^2 x$

$$\frac{dy}{dx} = \tan x \frac{d}{dx} (\cos^2 x) + \cos^2 x \frac{d}{dx} (\tan x)$$

$$= \tan x (2 \cos x) (-\sin x) + \cos^2 x \sec^2 x$$

$$= 1 - 2 \sin^2 x$$

DPP 2.4

Single Correct Answer Type

1. (a) $\frac{d}{dx} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 = \frac{d}{dx} \left[x + \frac{1}{x} + 1 \right] = 1 - \frac{1}{x^2}$

2. (a) Here $z = a - \frac{1}{y} \Rightarrow \frac{dz}{dy} = \frac{1}{y^2} = (a - z)^2$

3. (a) $y = x \sin x \Rightarrow \frac{dy}{dx} = x \cos x + \sin x \Rightarrow \frac{1}{y} \frac{dy}{dx} = \cot x + \frac{1}{x}$

4. (d) Since $\frac{dy}{dx} = -\sin(\sin x^2) \cdot \cos x^2 \cdot 2x$

Therefore, at $x = \sqrt{\frac{\pi}{2}}$, $\cos x^2 = \cos \frac{\pi}{2} = 0 \Rightarrow \frac{dy}{dx} = 0$

5. (d) $f(x) = x^2 - 3x$ and $f'(x) = 2x - 3$

But $f(x) = f'(x) \Rightarrow x^2 - 3x = 2x - 3 \Rightarrow x^2 - 5x + 3 = 0$

$$\therefore x = \frac{5 \pm \sqrt{25 - 12}}{2} = \frac{5 \pm \sqrt{13}}{2}$$

6. (c) Here $f'(x) = m = 1 \Rightarrow f'(0) = m = 1$ and $f(0) = c = 1$. Therefore $f(2) = 2 \times 1 + 1 = 3$.

7. (c) $x = y\sqrt{1-y^2}$

Differentiate with respect to x ,

$$1 = \frac{dy}{dx} \sqrt{1-y^2} + y \cdot \frac{1}{2\sqrt{1-y^2}} \cdot (-2y) \cdot \frac{dy}{dx}$$

$$\Rightarrow 1 = \frac{dy}{dx} \sqrt{1-y^2} - \frac{y^2}{\sqrt{1-y^2}} \cdot \frac{dy}{dx}$$

$$\Rightarrow 1 = \frac{dy}{dx} \left[\frac{1-y^2-y^2}{\sqrt{1-y^2}} \right] \Rightarrow 1 = \frac{dy}{dx} \left[\frac{1-2y^2}{\sqrt{1-y^2}} \right]$$

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{1-2y^2}$$

8. (c) $f(x) = \sqrt{ax} + \frac{a^2}{\sqrt{ax}}$, then

$$\Rightarrow f'(x) = \frac{\sqrt{a}}{2\sqrt{x}} + \frac{a^2}{\sqrt{a}} \left(-\frac{1}{2} x^{-3/2} \right)$$

$$\Rightarrow f'(x) = \frac{\sqrt{a}}{2\sqrt{x}} - \frac{a^2}{2\sqrt{a}} x^{-3/2}$$

$$\Rightarrow f'(a) = \frac{\sqrt{a}}{2\sqrt{a}} - \frac{a^2}{2\sqrt{a} \cdot a^{3/2}}$$

$$\Rightarrow f'(a) = \frac{1}{2} - \frac{a^2}{2a^2} = 0$$

9. (b) $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2at}{1+t^2}$

Differentiating with respect to t , we get

$$\frac{dx}{dt} = \frac{(1+t^2)(0-2t) - (1-t^2)(0+2t)}{(1+t^2)^2}$$

$$= -\frac{4t}{(1+t^2)^2}$$

and $\frac{dy}{dt} = \frac{(1+t^2)2a - 2at(2t)}{(1+t^2)^2} = \frac{2a(1-t^2)}{(1+t^2)^2}$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a(1-t^2)}{-2t} \therefore \frac{dy}{dx} = \frac{a(t^2-1)}{2t}$$

10. (c) $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$

Put $t = \tan \theta$ in both the equations, we get

$$x = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos 2\theta \text{ and } y = \frac{2 \tan \theta}{1 + \tan^2 \theta} = \sin 2\theta$$

Differentiating both the equations, we get

$$\frac{dx}{d\theta} = -2 \sin 2\theta \text{ and } \frac{dy}{d\theta} = 2 \cos 2\theta$$

Therefore $\frac{dy}{dx} = -\frac{\cos 2\theta}{\sin 2\theta} = -\frac{x}{y}$

$$11. (d) \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} \Rightarrow \frac{dy}{dx} = \frac{1}{t} = \frac{2a}{y}$$

$$\Rightarrow y \frac{dy}{dx} = 2a \Rightarrow y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-(dy/dx)^2}{y} = -\frac{1}{2at^3}$$

$$12. (d) x\sqrt{1+y} + y\sqrt{1+x} = 0 \Rightarrow x^2(1+y) = y^2(1+x) \quad \{x \neq y\}$$

$$\Rightarrow (x-y)(x+y+xy) = 0 \Rightarrow x+y+xy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+x)^2}$$

$$13. (a) \frac{dx}{dt} = -2 \sin t + 2 \sin 2t \text{ and } \frac{dy}{dt} = 2 \cos t - 2 \cos 2t$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos t - \cos 2t}{\sin 2t - \sin t}$$

$$\text{Put } t = \frac{\pi}{4}, \text{ we have } \left[\frac{dy}{dx}\right]_{t=\pi/4}$$

$$= \frac{\cos \pi/4 - \cos \pi/2}{\sin \pi/2 - \sin \pi/4} = \sqrt{2} + 1$$

$$14. (d) y = a \sin x + b \cos x$$

Differentiating with respect to x , we get

$$\frac{dy}{dx} = a \cos x - b \sin x$$

$$\text{Now } \left(\frac{dy}{dx}\right)^2 = (a \cos x - b \sin x)^2$$

$$= a^2 \cos^2 x + b^2 \sin^2 x - 2ab \sin x \cos x$$

$$\text{and } y^2 = (a \sin x + b \cos x)^2$$

$$= a^2 \sin^2 x + b^2 \cos^2 x + 2ab \sin x \cos x$$

$$\text{So, } \left(\frac{dy}{dx}\right)^2 + y^2 = a^2 (\sin^2 x + \cos^2 x) + b^2 (\sin^2 x + \cos^2 x)$$

$$\text{Hence } \left(\frac{dy}{dx}\right)^2 + y^2 = (a^2 + b^2) = \text{constant}$$

DPP 2.5

Single Correct Answer Type

$$1. (d) \text{ Given } f(x) = (x-1)(x-2)^2$$

$$f(x) = (x-1)(x^2 + 4 - 4x); f(x) = (x^3 - 5x^2 + 8x - 4)$$

$$\text{Now } f'(x) = 3x^2 - 10x + 8, f'(x) = 0$$

$$\Rightarrow 3x^2 - 10x + 8 = 0 \Rightarrow (3x-4)(x-2) = 0 \Rightarrow \frac{4}{3}, 2$$

$$\text{Now } f''(x) = 6x - 10$$

$$f''(4/3) = 6 \times 4/3 - 10 < 0$$

$$f''(2) = 12 - 10 > 0$$

Hence at $x = \frac{4}{3}$ the function will occupy maximum value.

$$\therefore \text{Maximum value} = f(4/3) = 4/27.$$

$$2. (c) 2x + 2y = 100 \Rightarrow x + y = 50$$

Let area of rectangle is A ; $\therefore A = xy \Rightarrow y = \frac{A}{x}$

$$\text{Put in (i), we have } x + \frac{A}{x} = 50 \Rightarrow A = 50x - x^2$$

$$\Rightarrow \frac{dA}{dx} = 50 - 2x$$

$$\text{For maximum area } \frac{dA}{dx} = 0$$

$$\therefore 50 - 2x = 0 \Rightarrow x = 25 \text{ and } y = 25$$

Hence adjacent sides are 25 and 25 cm.

3. (c) We know that perimeter of a rectangle $S = 2(x+y)$, where x and y are adjacent sides

$$\Rightarrow y = \frac{S-2x}{2}. \text{ Now area of rectangle,}$$

$$A = xy = \frac{x}{2}(S-2x) = \frac{1}{2}(Sx-2x^2)$$

Differentiating w.r.t. x of A , we get

$$\frac{dA}{dx} = \frac{1}{2}(S-4x) = 0 \therefore x = \frac{S}{4} \text{ and } y = \frac{S}{4}$$

$$\text{Again } \frac{d^2A}{dx^2} = -ve$$

Hence the area of rectangle will be maximum when rectangle is a square.

$$4. (c) x + y = 10; \therefore y = 10 - x$$

$$\text{Now } f(x) = xy = x(10-x) = 10x - x^2$$

$$\therefore f'(x) = 10 - 2x$$

$$\text{For maximum value of } f(x), f'(x) = 0$$

$$\therefore x = 5 \text{ and } y = 5$$

$$\text{So maximum value of } xy = 5 \times 5 = 25.$$

5. (a) Suppose that two numbers are x and y .

$$x + y = s \Rightarrow y = s - x$$

$$\text{Then } f(x) = xy = x(s-x) = xs - x^2$$

$$\therefore f'(x) = s - 2x$$

$$f'(x) = 0 \text{ for maximum value of } f(x)$$

$$\therefore x = \frac{s}{2} \text{ and } y = \frac{s}{2}$$

Thus each number is half of the sum.

6. (a) Let the first number be $3-x$, then second will be x .

Accordingly, we have to maximize $(3-x)x^2$

$$\text{Let } f(x) = (3-x)x^2 = 3x^2 - x^3 \Rightarrow f'(x) = 6x - 3x^2$$

$$\therefore f'(x) = 0 \Rightarrow x = 0, 2$$

$$\text{Also } f''(x) = 6 - 6x. \text{ Obviously, } f''(2) = -6 < 0.$$

$$\text{Therefore, required max. value} = (3-2) \cdot 2^2 = 4.$$

(b) Given $2(a+b) = 36$, $a+b = 18$
Area of rectangle $= ab = a(18-a)$

$$A = 18a - a^2, \therefore \frac{dA}{da} = 18 - 2a$$

Put $\frac{dA}{da} = 0$ i.e., $18 - 2a = 0 \Rightarrow a = 9$; $b = 9$.

(d) Let $x+y = 20 \Rightarrow y = 20-x$

and $x^3 \cdot y^2 = z \Rightarrow z = x^3 \cdot y^2$

$$z = x^3(20-x)^2 \Rightarrow z = 400x^3 + x^5 - 40x^4$$

$$\frac{dz}{dx} = 1200x^2 + 5x^4 - 160x^3$$

Now $\frac{dz}{dx} = 0$, then $x = 12, 20$

Now $\frac{d^2z}{dx^2} = 2400x + 16x^3 - 480x^2; \left(\frac{d^2z}{dx^2}\right)_{x=12} = -ive$

Hence $x = 12$ is the point of maxima

$\therefore x = 12, y = 8$.

(a) $xy = 1 \Rightarrow y = \frac{1}{x}$ and let $z = x + y$

$$z = x + \frac{1}{x} \Rightarrow \frac{dz}{dx} = 1 - \frac{1}{x^2}$$

Now $\frac{dz}{dx} = 0 \Rightarrow 1 - \frac{1}{x^2} = 0$

$$x = -1, +1 \text{ and } \frac{d^2z}{dx^2} = \frac{2}{x^3}$$

$$\left(\frac{d^2z}{dx^2}\right)_{x=1} = \frac{2}{1} = 2 = +ive$$

$\therefore$ Hence $x = 1$ is point of minima and $x = 1$ and $y = 1$

$\therefore$ Minimum value $x + y = 2$.

(b) $x + y = 20$ and $z = xy^3$

$$\Rightarrow z = y^3(20-y) = 20y^3 - y^4$$

$$\Rightarrow \frac{dz}{dy} = 60y^2 - 4y^3 = 0 \Rightarrow 4y^2(15-y) = 0$$

So, either $y = 0$, or $y = 15$

Now, $\frac{d^2z}{dy^2} = 120y - 12y^2$; $\therefore$ At $y = 0$, $\frac{d^2z}{dy^2} > 0$

$\therefore y = 0$ is the point of minima and at $y = 15$, $\frac{d^2z}{dy^2} < 0$

$\therefore y = 15$ is the point of maxima.

Hence the required parts is $(5, 15)$.

11. (c) Let $y = x^3 - 18x^2 + 96x \Rightarrow \frac{dy}{dx} = 3x^2 - 36x + 96 = 0$

$$\therefore x^2 - 12x + 32 = 0 \Rightarrow (x-4)(x-8) = 0, x = 4, 8$$

Now, $\frac{d^2y}{dx^2} = 6x - 36$

At $x = 4$, $\frac{d^2y}{dx^2} = 24 - 36 = -12 < 0$

$\therefore$ At $x = 4$ function will be maximum

and $[f(4)]_{\max} = 64 - 288 + 384 = 160$

At $x = 8$, $\frac{d^2y}{dx^2} = 48 - 36 = 12 > 0$

$\therefore$ At $x = 8$, function will be minimum and $[f(8)]_{\min} = 128$.

12. (c) $y = \sin x(1 + \cos x) = \sin x + \frac{1}{2} \sin 2x$

$$\therefore \frac{dy}{dx} = \cos x + \cos 2x \text{ and } \frac{d^2y}{dx^2} = -\sin x - 2 \sin 2x$$

On putting $\frac{dy}{dx} = 0$, $\cos x + \cos 2x = 0$

$$\Rightarrow \cos x = -\cos 2x = \cos(\pi - 2x) \Rightarrow x = \pi - 2x$$

$$\therefore x = \frac{\pi}{3}; \therefore \left(\frac{d^2y}{dx^2}\right)_{x=\pi/3} = -\sin\left(\frac{1}{3}\pi\right) - 2\sin\left(\frac{2}{3}\pi\right)$$

$$= \frac{-\sqrt{3}}{2} - 2 \cdot \frac{\sqrt{3}}{2} = \frac{-3\sqrt{3}}{2}, \text{ which is negative.}$$

$\therefore$ At $x = \frac{\pi}{3}$ the function is maximum.

13. (b) $f(x) = 2x^3 - 15x^2 + 36x + 4 \Rightarrow f'(x) = 6x^2 - 30x + 36$ (i)

We know that for its maximum value

$$f'(x) = 0, 6x^2 - 30x + 36 = 0 \Rightarrow (x-2)(x-3) = 0 \Rightarrow x = 2, 3.$$

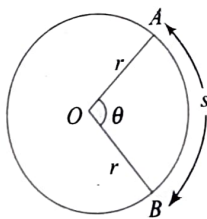
Again differentiating equation (i), we get $f''(x) = 12x - 30$

$$\Rightarrow f''(2) = 24 - 30 = -6 < 0.$$

Therefore $f(x)$ is maximum at $x = 2$.

$\therefore$ Given function has one maximum and one minimum.

14. (d)



Perimeter of a sector $= p$. Let AOB be the sector with radius r . If angle of the sector be θ radians, then area of sector (A) =

$$\frac{1}{2}r^2\theta$$
 (i)

$$\text{Length of arc}(s) = r\theta \text{ or } \theta = \frac{s}{r}.$$

Therefore perimeter of the sector

$$= (p) = r + s + r = 2r + s$$
 (ii)

$$\text{Substituting } \theta = \frac{s}{r} \text{ in (i), } A = \left(\frac{1}{2}r^2\right)\left(\frac{s}{r}\right) = \frac{1}{2}rs$$

$$\Rightarrow s = \frac{2A}{r}. \text{ Now substituting the value of } s \text{ in (ii), we get}$$

$$p = 2r + \left(\frac{2A}{r}\right) \text{ or } 2A = pr - 2r^2. \text{ Differentiating with respect}$$

to r , we get $2\frac{dA}{dr} = p - 4r$. We know that for the maximum

value of area $\frac{dA}{dr} = 0$ or $p - 4r = 0$ or $r = \frac{p}{4}$.

15. (a) Let $y = f(x) = \left(x^2 + \frac{250}{x}\right)$, $\therefore \frac{dy}{dx} = f'(x) = 2x - \frac{250}{x^2}$

Put $f'(x) = 0 \Rightarrow 2x^3 - 250 = 0 \Rightarrow x^3 = 125 \Rightarrow x = 5$

Again, $\frac{d^2y}{dx^2} = f''(x) = 2 + \frac{500}{x^3}$. Now $f''(5) = 2 + \frac{500}{125} > 0$

Hence at $x = 5$. The function will be minimum.

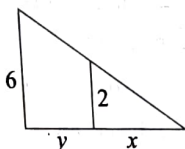
Minimum value $f(5) = 25 + 50 = 75$.

DPP 2.6

Single Correct Answer Type

1. (b) $\frac{dy}{dt} = 5, \frac{dx}{dt} = ?$

From figure, $\frac{x}{2} = \frac{x+y}{6} \Rightarrow 4x = 2y \Rightarrow x = \frac{1}{2}y$



Hence $\frac{dx}{dt} = \frac{1}{2} \frac{dy}{dt} = \frac{5}{2}$ metre/hour

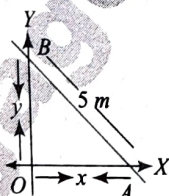
2. (a) According to fig. $x^2 + y^2 = 25$

Differentiate (i) w.r.t. t , we get

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

Here $x = 4$ and $\frac{dx}{dt} = 1.5$

From (i), $4^2 + y^2 = 25 \Rightarrow y = 3$



$\therefore$ From (ii), $2(4)(1.5) + 2(3) \frac{dy}{dt} = 0$

So, $\frac{dy}{dt} = -2$ m/sec

Hence, length of the highest point decreases at the rate of 2 m/sec.

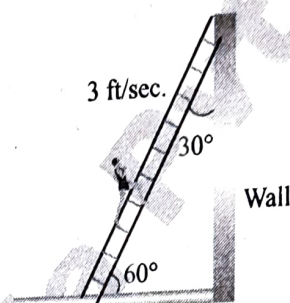
3. (a) Given the rate of increasing the radius $= \frac{dr}{dt} = 3.5$ cm/sec and $r = 10$ cm

Area of circle $= \pi r^2$, $A = \pi r^2$

$\Rightarrow \frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} \Rightarrow \frac{dA}{dt} = 2\pi \times 10 \times 3.5$

$\Rightarrow \frac{dA}{dt} = 220$ cm²/sec

4. (b)



His rate of approaching the wall $= 3 \times \cos 60^\circ = \frac{3}{2}$ ft/sec.

5. (b) $\frac{da}{dt} = 60$ cm/sec, where a is edge and t is time.

$V = a^3$.

$\therefore \frac{dV}{dt} = 3a^2 \frac{da}{dt} = 3a^2 \times 60 = 180a^2 = 180 \times (90)^2$
 $= 1458000$ cm³/sec.

6. (c) Perimeter $P = 2\pi r$

$\frac{dP}{dt} = 2\pi \cdot \frac{dr}{dt}$ and $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} \Rightarrow \frac{dA}{dt} = \frac{dP}{dt} \times r \Rightarrow \frac{dA}{dt} \propto r$

7. (c) If x is the length of each side of an equilateral triangle and A is

its area, then $A = \frac{\sqrt{3}}{4} x^2 \Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{4} 2x \frac{dx}{dt}$

Here, $x = 10$ cm and $\frac{dx}{dt} = 2$ cm/sec

$\Rightarrow A = 10\sqrt{3}$ sq. unit per sec.

8. (d) $V = \frac{4}{3} \pi (x + 10)^3$ where x is thickness of ice.

$\therefore \frac{dV}{dt} = 4\pi(10 + x)^2 \frac{dx}{dt}$

$\therefore$ At $x = 5$, $\left(\frac{dx}{dt}\right) = \frac{1}{18\pi}$ cm/min

9. (b) Volume $= V = \frac{4}{3} \pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$, at $r = 7$ cm

35 cc/min $= 4\pi(7)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{5}{28\pi}$

Surface area, $S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt} = \frac{8\pi \cdot 7 \cdot 5}{28\pi} = 10$ cm²/min

$$p(t) = 1000 + \frac{1000t}{100 + t^2}$$

$$\frac{dp}{dt} = \frac{(100 + t^2)1000 - 1000t \cdot 2t}{(100 + t^2)^2} = \frac{1000(100 - t^2)}{(100 + t^2)^2}$$

$$\text{For extremum, } \frac{dp}{dt} = 0 \Rightarrow t = 10$$

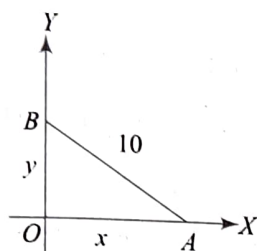
$$\text{Now } \left. \frac{dp}{dt} \right|_{t < 10} > 0 \text{ and } \left. \frac{dp}{dt} \right|_{t > 10} < 0$$

$\therefore$ At $t = 10$, $\frac{dp}{dt}$ change from positive to negative.

$\therefore$ p is maximum at $t = 10$.

$$\therefore p_{\max} = p(10) = 1000 + \frac{1000 \cdot 10}{100 + 10^2} = 1050$$

$$(b) \text{ We have } x^2 + y^2 = 10^2$$



$$\Rightarrow 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \Rightarrow x \cdot 3 + y \cdot (-4) = 0$$

$$x = \frac{4}{3}y. \text{ Thus, } \left(\frac{4}{3}y\right)^2 + y^2 = 10^2 \Rightarrow y = 6 \text{ m}$$

DPP 2.7

Subjective Type

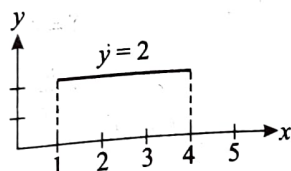
1. (a) $\int \frac{3}{2} \sqrt{x} dx = \frac{3}{2} \cdot \frac{x^{3/2}}{3/2} + C = \sqrt{x^3} + C$
- (b) $\int \frac{3}{2\sqrt{x}} dx = \frac{3}{2} \cdot \frac{x^{1/2}}{1/2} + C = 3\sqrt{x} + C$
- (c) $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx = \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C = \frac{2}{3} \sqrt[3]{x} + 2\sqrt{x} + C$

$$2. u = 2t - 4$$

$$\frac{dx}{dt} = 2$$

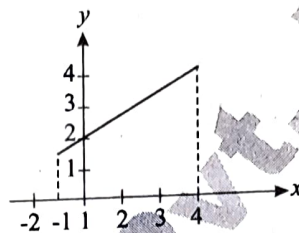
$$\int \frac{u^{-4} dx}{2} = \frac{1}{2} \left(\frac{u^{-3}}{-3} \right) + C$$

3. (a) The graph of the integrand is the horizontal line $y = 2$, so the region is a rectangle of height 2 extending over the interval from 1 to 4.



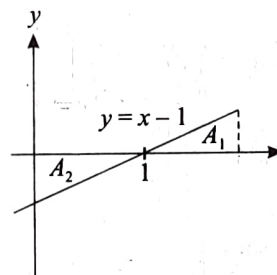
$$\int_1^4 2 dx = (\text{area of rectangle}) = 2(3) = 6$$

- (b) The graph of the integrand is the line $y = x + 2$, so the region is a trapezoid whose base extends from $x = -1$ to $x = 2$ [Fig. (b)]. Thus,



$$\int_{-1}^2 (x+2) dx = (\text{area of trapezoid}) = \frac{1}{2}(1+4)(3) = \frac{15}{2}$$

- (c) The graph of $y = x - 1$ is shown in figure and we leave it for you to verify that the shaded triangular regions both have area $\frac{1}{2}$. Over the interval $[0, 2]$ the net signed is $A_1 - A_2 = \frac{1}{2} - \frac{1}{2} = 0$, and over the interval $[0, 1]$ the net signed area is $-A_2 = -\frac{1}{2}$. Thus,



$$\int_0^2 (x-1) dx = 0 \text{ and } \int_0^1 (x-1) dx = -\frac{1}{2}$$

4. (a) Since $\cos x \geq 0$ over the interval $[0, \pi/2]$, the area A under the curve is

$$A = \int_0^{\pi/2} \cos x dx = [\sin x]_0^{\pi/2} = \sin \frac{\pi}{2} - \sin 0 = 1$$

- (b) The given integral can be interpreted as the signed area between the graph of $y = \cos x$ and the interval $[0, \pi]$. The graph in figure suggests that over the interval $[0, \pi]$ the portion of area above the x -axis is the same as the portion of area below the x -axis, so we conjecture that the signed area is zero; this implies that the value of the integral is zero. This is confirmed by the computations.

$$\int_0^{\pi} \cos x dx = [\sin x]_0^{\pi} = \sin \pi - \sin 0 = 0$$

Single Correct Answer Type

5. (d) $\int \sqrt{1 + \sin x} dx = \int \sqrt{\left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2} dx$
 $= \int \sin \frac{x}{2} dx + \int \cos \frac{x}{2} dx = -2 \cos \frac{x}{2} + 2 \sin \frac{x}{2} + c$
 $= -2 \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right) + c = -2 \sqrt{1 - \sin x} + c$

$$6. (a) \int \left(2 \sin x + \frac{1}{x} \right) dx = -2 \cos x + \log x + c$$

$$7. (a) \quad I = \int \sqrt{1 + \cos x} \, dx = \int \sqrt{2 \cos^2(x/2)} \, dx \\ I = \sqrt{2} \int \cos(x/2) \, dx = 2\sqrt{2} \sin(x/2) + c$$

$$8. (d) \quad I = \int 2 \sin x \cdot \cos x \, dx = \int \sin 2x \, dx \\ = -\frac{\cos 2x}{2} + c = -\frac{(1 - 2 \sin^2 x)}{2} + c \\ = -\frac{1}{2} + \sin^2 x + c = \sin^2 x + c$$

$$9. (b) \quad y = \sin(2x + 3)$$

$$\text{Let } 2x + 3 = t \Rightarrow 2dx = dt$$

$$\Rightarrow \int y \, dx = \int \sin(2x + 3) \, dx \\ = \int \sin t \cdot \frac{dt}{2} = \frac{1}{2} (-\cos(t)) = -\frac{\cos(2x + 3)}{2} + C$$

$$10. (a) \quad \int 2 \sin(x) \, dx = -2 \cos x + C$$

$$11. (b) \quad y = x^2 \sin x^3, \quad \int_0^1 y \, dx = \int_0^1 x^2 \, dx = \left[\frac{x^3}{3} \right]_0^1 = \frac{1}{3}$$

$$12. (b) \quad \text{Area} \int y \, dx = \int_0^2 x^2 \, dx = \left[\frac{x^3}{3} \right]_0^2 = \frac{8}{3}$$

$$13. (b) \quad \int_0^{\pi/2} \cos 3t \, dt = \left[\frac{\sin 3t}{3} \right]_0^{\pi/2} \\ = \frac{1}{3} [-1 - 0] = -\frac{1}{3}$$

$$14. (a) \quad \int \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right)^2 dx \\ = \int \left\{ \cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} - 2 \sin \frac{x}{2} \cos \frac{x}{2} \right\} dx \\ = \int (1 - \sin x) \, dx = x + \cos x + c$$

Comprehension Type

$$15. (c) \quad a = 3t^2 + 2t + 1$$

$$\int_0^t dv = \int_0^t (3t^2 + 2t + 1) \, dt$$

$$v = t^3 + t^2 + t$$

$$16. (b) \quad V(t - 0) = 0$$

$$V_{t=3} = (3)^3 + (3)^2 + 3$$

$$= 27 + 9 + 3$$

$$= 39$$

$$\Delta V = 39 - 0 = 39 \, \text{m/s}^2$$

$$17. (b) \quad \int_0^s dS = \int_0^2 (t^3 + t^2 + t) \, dt$$

$$S = \left[\frac{t^4}{4} + \frac{t^3}{3} + \frac{t^2}{2} \right]_0^2$$

$$S = 4 + \frac{8}{3} + 2$$

$$S = \frac{12 + 8 + 6}{3} = \frac{26}{3}$$

Fill in the Blanks

18. This statement has no meaning.

$$19. \quad \int x^3 \, dx = \frac{x^4}{4} + C$$

$$20. \quad \int \left(\frac{1}{5} - \frac{2}{x^3} + 2x \right) dx = \frac{x}{5} + \frac{1}{x^2} + x^2 + C$$

$$21. \quad y = 3xv + 2x + 4$$

$$\int (3x^2 + 2x + 4) \, dx$$

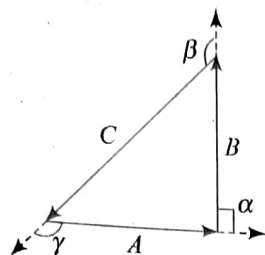
$$x^3 + x^2 + 4x + c$$

$$22. \quad \int \frac{dx}{ax + b} = \frac{1}{a} \log_e(ax + b) + C$$

CHAPTER 3

Single Correct Answer Type

1. (d) From polygon law, three vectors having summation zero should form a closed polygon (Triangle). Since the two vectors are having same magnitude and the third vector is $\sqrt{2}$ times that of either of two having equal magnitude. That is, the triangle should be right angled triangle



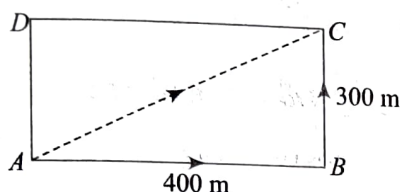
Angle between A and B, $\alpha = 90^\circ$

Angle between B and C, $\beta = 135^\circ$

Angle between A and C, $\gamma = 135^\circ$

2. (b)

Displacement $\overline{AC} = \overline{AB} + \overline{BC}$



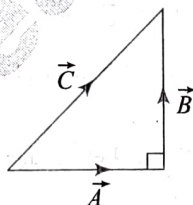
$$AC = \sqrt{(AB)^2 + (BC)^2} = \sqrt{(400)^2 + (300)^2} = 500 \text{ m}$$

Distance = $AB + BC = 400 + 300 = 700 \text{ m}$

3. (d) Here all the three force will not keep the particle in equilibrium so the net force will not be zero and the particle will move with an acceleration.

4. (a) $C = \sqrt{A^2 + B^2}$
 $= \sqrt{3^2 + 4^2} = 5$

$\therefore$ Angle between $\vec{A}$ and $\vec{B}$ is $\frac{\pi}{2}$

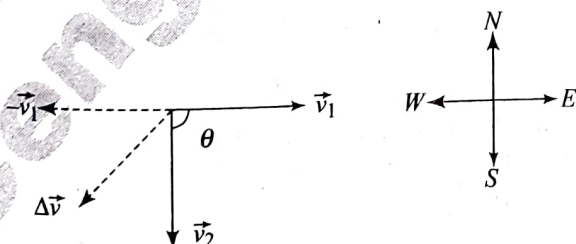


5. (d) Total angle = $100 \times \frac{\pi}{50} = 2\pi$

So all the force will pass through one point and all forces will be balanced, i.e., their resultant will be zero.

6. (a) If the angle between all forces which are equal and lying in one plane are equal, then resultant force will be zero.

7. (d)



If the magnitude of vector remains same, only direction change by θ then

$$\Delta \vec{v} = \vec{v}_2 - \vec{v}_1, \Delta \vec{v} = \vec{v}_2 + (-\vec{v}_1)$$

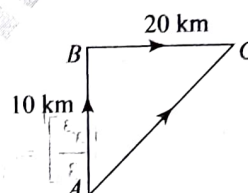
Magnitude of change in vector $|\Delta \vec{v}| = 2v \sin\left(\frac{\theta}{2}\right)$

$$|\Delta \vec{v}| = 2 \times 10 \times \sin\left(\frac{90^\circ}{2}\right) = 10\sqrt{2} = 14.14 \text{ m/s}$$

Direction is south-west as shown in figure.

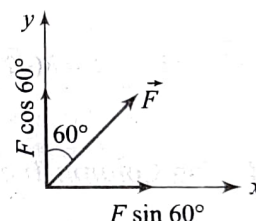
8. (a) $\overline{AC} = \overline{AB} + \overline{BC}$

$$\begin{aligned} AC &= \sqrt{(AB)^2 + (BC)^2} \\ &= \sqrt{(10)^2 + (20)^2} \\ &= \sqrt{100 + 400} = \sqrt{500} = 22.36 \text{ km} \end{aligned}$$



9. (c) If vectors are of equal magnitude then two vectors can give zero resultant, if they work in opposite direction. But if the vectors are of different magnitudes, then minimum three vectors are required to give zero resultant.

10. (d)



The component of force in vertical direction

$$= F \cos \theta = F \cos 60^\circ = 5 \times \frac{1}{2} = 2.5 \text{ N}$$

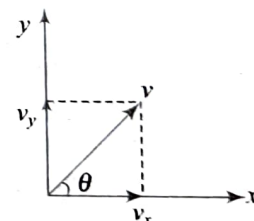
11. (a) $v_y = 20$ and $v_x = 10$

$$\therefore \text{velocity } \vec{v} = 10\hat{i} + 20\hat{j}$$

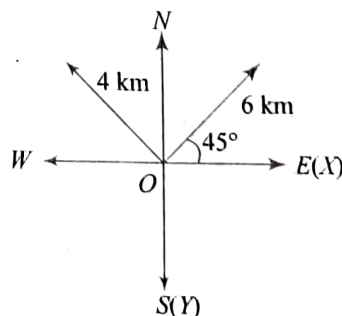
Direction of velocity with x axis

$$\tan \theta = \frac{v_y}{v_x} = \frac{20}{10} = 2$$

$$\therefore \theta = \tan^{-1}(2)$$



12. (c)



Net movement along x-direction

$$S_x = (6 - 4) \cos 45^\circ \hat{i} = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2} \text{ km}$$

Net movement along y-direction

$$S_y = (6 + 4) \sin 45^\circ \hat{j} = 10 \times \frac{1}{\sqrt{2}} = 5\sqrt{2} \text{ km}$$

Net movement from starting point

$$|\vec{S}| = \sqrt{S_x^2 + S_y^2} = \sqrt{(\sqrt{2})^2 + (5\sqrt{2})^2} = \sqrt{52} \text{ km}$$

Angle which makes with the east direction

$$\tan \theta = \frac{\text{Y-component}}{\text{X-component}} = \frac{5\sqrt{2}}{\sqrt{2}} \therefore \theta = \tan^{-1}(5)$$

13. (b) From the figure
- $|\vec{OA}| = a$
- and
- $|\vec{OB}| = a$

Also from triangle rule $\vec{OB} - \vec{OA} = \vec{AB} = \Delta \vec{a}$

$$\Rightarrow |\Delta \vec{a}| = AB$$

Using angle = $\frac{\text{arc}}{\text{radius}}$

$$\Rightarrow AB = a \cdot d\theta$$

$$\text{So } |\Delta \vec{a}| = a d\theta$$

 Δa means change in magnitude of vector, i.e., $|\vec{OB}| - |\vec{OA}|$

$$\Rightarrow a - a = 0$$

$$\text{So } \Delta a = 0$$

14. (b)
- $R_{\text{net}} = R + \sqrt{R^2 + R^2} = R + \sqrt{2}R = R(\sqrt{2} + 1)$

Matching Column Type

15. A-(ii), B-(iii), C-(i), D-(iv)

DPP 3.2

Single Correct Answer Type

1. (b)
- $R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$
-
- By substituting,
- $A = F$
- ,
- $B = F$
- and
- $R = F$
- we get

$$\cos \theta = \frac{1}{2} \therefore \theta = 120^\circ$$

2. (c)
- $\vec{R}_1 = \vec{A} + \vec{B}$
- ,
- $\vec{R}_2 = \vec{A} - \vec{B}$
-
- $R_1^2 + R_2^2 = (\sqrt{A^2 + B^2})^2 + (\sqrt{A^2 + B^2})^2 = 2(A^2 + B^2)$

3. (c)
- $F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos 90^\circ} = \sqrt{F_1^2 + F_2^2}$

4. (a)
- $A + B = 16$
- (given)

$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta} = \tan 90^\circ$$

$$\therefore A + B \cos \theta = 0 \Rightarrow \cos \theta = \frac{-A}{B}$$

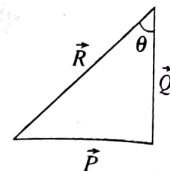
$$8 = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

By solving eq. (i), (ii) and (iii) we get $A = 6 \text{ N}$, $B = 10 \text{ N}$

5. (c)
- $|\vec{P}| = 5$
- ,
- $|\vec{Q}| = 12$
- and
- $|\vec{R}| = 13$

$$\cos \theta = \frac{Q}{R} = \frac{12}{13}$$

$$\therefore \theta = \cos^{-1}\left(\frac{12}{13}\right)$$



6. (b)
- $\frac{B}{2} = \sqrt{A^2 + B^2 + 2AB \cos \theta}$

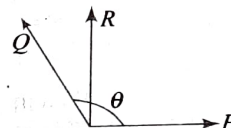
$$\therefore \tan 90^\circ = \frac{B \sin \theta}{A + B \cos \theta} \Rightarrow A + B \cos \theta = 0$$

$$\therefore \cos \theta = -\frac{A}{B}$$

$$\text{Hence, from (i)} \quad \frac{B^2}{4} = A^2 + B^2 - 2A^2 \Rightarrow A = \sqrt{3} \frac{B}{2}$$

$$\Rightarrow \cos \theta = -\frac{A}{B} = -\frac{\sqrt{3}}{2} \therefore \theta = 150^\circ$$

7. (b)



$$\Rightarrow \tan 90^\circ = \frac{Q \sin \theta}{P + Q \cos \theta} \Rightarrow P + Q \cos \theta = 0$$

$$\cos \theta = \frac{-P}{Q} \therefore \theta = \cos^{-1}\left(\frac{-P}{Q}\right)$$

8. (b) Let
- $\hat{n}_1$
- and
- $\hat{n}_2$
- are the two unit vectors, then the sum is

$$\vec{n}_s = \hat{n}_1 + \hat{n}_2 \text{ or } n_s^2 = n_1^2 + n_2^2 + 2n_1n_2 \cos \theta$$

$$= 1 + 1 + 2 \cos \theta$$

Since it is given that n_s is also a unit vector, therefore

$$1 = 1 + 1 + 2 \cos \theta \Rightarrow \cos \theta = -\frac{1}{2} \therefore \theta = 120^\circ$$

Now the difference vector is

$$\vec{n}_d = \hat{n}_1 - \hat{n}_2 \text{ or } n_d^2 = n_1^2 + n_2^2 - 2n_1n_2 \cos \theta = 1 + 1 - 2 \cos(120^\circ)$$

$$\therefore n_d^2 = 2 - 2(-1/2) = 2 + 1 = 3 \Rightarrow n_d = \sqrt{3}$$

9. (a) According to problem
- $P + Q = 3$
- and
- $P - Q = 1$
-
- By solving we get
- $P = 2$
- and
- $Q = 1$

$$\therefore \frac{P}{Q} = 2 \Rightarrow P = 2Q$$

10. (d)
- $A = 3 \text{ N}$
- ,
- $B = 2 \text{ N}$
- , then
- $R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$

$$R = \sqrt{9 + 4 + 12 \cos \theta}$$

Now $A = 6 \text{ N}$, $B = 2 \text{ N}$, then

$$2R = \sqrt{36 + 4 + 24 \cos \theta}$$

from (i) and (ii) we get $\cos \theta = -\frac{1}{2} \therefore \theta = 120^\circ$

11. (c) Resultant of two vectors $\vec{A}$ and $\vec{B}$ can be given by $\vec{R} = \vec{A} + \vec{B}$

$$\vec{R} = \vec{A} + \vec{B}$$

$$|\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

If $\theta = 0^\circ$, then $|\vec{R}| = A + B = |\vec{A}| + |\vec{B}|$

12. (d) $R_{\max} = A + B = 17$ when $\theta = 0^\circ$

$$R_{\min} = A - B = 7 \text{ when } \theta = 180^\circ$$

by solving we get $A = 12$ and $B = 5$

Now when $\theta = 90^\circ$ then $R = \sqrt{A^2 + B^2}$

$$\Rightarrow R = \sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13$$

13. (c) $R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$

If $A = B = P$ and $\theta = 120^\circ$ then $R = P$

14. (c) Let P be the smaller force and Q be the greater force, then according to problem –

$$P + Q = 18$$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta} = 12$$

$$\tan \phi = \frac{Q \sin \theta}{P + Q \cos \theta} = \tan 90 = \infty$$

$$\therefore P + Q \cos \theta = 0 \quad \text{(iii)}$$

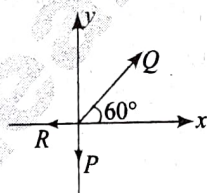
By solving (i), (ii) and (iii) we will get $P = 5$, and $Q = 13$

15. (d) The particle is at rest under action of forces P , Q and R .

$$\therefore Q \sin 60^\circ = P \text{ and } Q \cos 60^\circ = R$$

$$\Rightarrow \frac{2}{\sqrt{3}} P = Q \text{ and } 2R = Q$$

$$\Rightarrow P : Q : R = \sqrt{3} : 2 : 1$$



DPP 3.3

Single Correct Answer Type

1. (c) Displacement vector $\vec{r} = \Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k}$
 $= (3 - 2)\hat{i} + (4 - 3)\hat{j} + (5 - 5)\hat{k} = \hat{i} + \hat{j}$

2. (b) $\vec{A} - 2\vec{B} + 3\vec{C} = (2\hat{i} + \hat{j}) - 2(3\hat{j} - \hat{k}) + 3(6\hat{i} - 2\hat{k})$
 $= 2\hat{i} + \hat{j} - 6\hat{j} + 2\hat{k} + 18\hat{i} - 6\hat{k} = 20\hat{i} - 5\hat{j} - 4\hat{k}$

3. (d) $x = 0$ means y-axis $\Rightarrow \vec{F}_1 = \hat{j}$

$$y = 0 \text{ means } x\text{-axis} \Rightarrow \vec{F}_2 = 2\hat{i}$$

$$\text{So resultant } \vec{F} = \vec{F}_1 + \vec{F}_2 = 2\hat{i} + \hat{j}$$

- (i) 4. (b) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

$$= (-4\hat{i} + 5\hat{j} - 3\hat{k} + 2\hat{i}) + (-5\hat{j} + 8\hat{j} + 4\hat{j} - 3\hat{j})$$

$$+ (5\hat{k} + 6\hat{k} - 7\hat{k} - 2\hat{k}) = 4\hat{j} + 2\hat{k}$$

$\therefore$ the particle will move in y-z plane.

5. (d) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 \Rightarrow 4\hat{i} + 6\hat{j} + \vec{F}_3 = 0$

$$\therefore \vec{F}_3 = -4\hat{i} - 6\hat{j}$$

6. (b) Unit vector along y axis $\hat{j}$ so the required vector
 $= \hat{j} - [(i - 3\hat{j} + 2\hat{k}) + (3\hat{i} + 6\hat{j} - 7\hat{k})] = -4\hat{i} - 2\hat{j} + 5\hat{k}$

7. (d) $|\vec{B}| = \sqrt{7^2 + (24)^2} = \sqrt{625} = 25$

Unit vector in the direction of $\vec{A}$ will be $\hat{A} = \frac{3\hat{i} + 4\hat{j}}{5}$

$$\text{So required vector} = 25 \left(\frac{3\hat{i} + 4\hat{j}}{5} \right) = 15\hat{i} + 20\hat{j}$$

8. (a) Resultant of vectors $\vec{A}$ and $\vec{B}$

$$\vec{R} = \vec{A} + \vec{B} = 4\hat{i} + 3\hat{j} + 6\hat{k} - \hat{i} + 3\hat{j} - 8\hat{k}$$

$$\vec{R} = 3\hat{i} + 6\hat{j} - 2\hat{k}$$

$$\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{3^2 + 6^2 + (-2)^2}} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$$

9. (a) $\vec{r} = 3t^2\hat{i} + 4t^2\hat{j} + 7\hat{k}$

$$\text{at } t = 0, \vec{r}_1 = 7\hat{k}$$

$$\text{at } t = 10 \text{ sec, } \vec{r}_2 = 300\hat{i} + 400\hat{j} + 7\hat{k}$$

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1 = 300\hat{i} + 400\hat{j}$$

$$|\Delta \vec{r}| = |\vec{r}_2 - \vec{r}_1| = \sqrt{(300)^2 + (400)^2} = 500 \text{ m}$$

10. (b) Resultant of vectors $\vec{A}$ and $\vec{B}$

$$\vec{R} = \vec{A} + \vec{B} = 4\hat{i} - 3\hat{j} + 8\hat{i} + 8\hat{j} = 12\hat{i} + 5\hat{j}$$

$$\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{12\hat{i} + 5\hat{j}}{\sqrt{(12)^2 + (5)^2}} = \frac{12\hat{i} + 5\hat{j}}{13}$$

11. (c) $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{B} = \hat{i} - 3\hat{j} + 5\hat{k}$, $\vec{C} = 2\hat{i} - \hat{j} + 4\hat{k}$

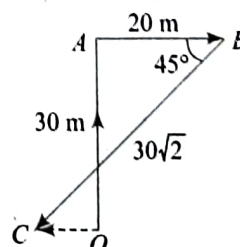
$$|\vec{A}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$$

$$|\vec{B}| = \sqrt{1^2 + (-3)^2 + 5^2} = \sqrt{1 + 9 + 25} = \sqrt{35}$$

$$|\vec{C}| = \sqrt{2^2 + 1^2 + (-4)^2} = \sqrt{4 + 1 + 16} = \sqrt{21}$$

As $B = \sqrt{A^2 + C^2}$ therefore ABC will be right angled triangle.

12. (c) From figure, $\vec{OA} = 0\hat{i} + 30\hat{j}$, $\vec{AB} = 20\hat{i} + 0\hat{j}$



$$\overrightarrow{BC} = -30\sqrt{2} \cos 45^\circ \hat{i} - 30\sqrt{2} \sin 45^\circ \hat{j} = -30\hat{i} - 30\hat{j}$$

$$\therefore \text{Net displacement, } \overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{BC} = -10\hat{j} + 0\hat{i}$$

$$|\overrightarrow{OC}| = 10 \text{ m}$$

DPP 3.4

Single Correct Answer Type

1. (c) $\vec{R} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}$

Comparing the given vector with $R = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$

$$R_x = 1, R_y = 1, R_z = \sqrt{2} \text{ and } |\vec{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = 2$$

$$\cos \alpha = \frac{R_x}{R} = \frac{1}{2} \Rightarrow \alpha = 60^\circ$$

$$\cos \beta = \frac{R_y}{R} = \frac{1}{2} \Rightarrow \beta = 60^\circ$$

$$\cos \gamma = \frac{R_z}{R} = \frac{1}{\sqrt{2}} \Rightarrow \gamma = 45^\circ$$

2. (c) $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$
 $= 1 - \cos^2 \alpha + 1 - \cos^2 \beta + 1 - \cos^2 \gamma$
 $= 3 - (\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma) = 3 - 1 = 2$

3. (b) Let $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -4\hat{i} - 6\hat{j} + \lambda\hat{k}$

$\vec{A}$ and $\vec{B}$ are parallel to each other

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} \text{ i.e. } \frac{2}{-4} = \frac{3}{-6} = \frac{-1}{\lambda} \Rightarrow \lambda = 2$$

4. (a) $\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} = \frac{(3\hat{i} + 4\hat{j} + 5\hat{k})(3\hat{i} + 4\hat{j} - 5\hat{k})}{\sqrt{9+16+25}\sqrt{9+16+25}}$
 $= \frac{9+16-25}{50} = 0$

$$\Rightarrow \cos \theta = 0, \therefore \theta = 90^\circ$$

5. (c) Given vectors can be rewritten as $\vec{A} = 2\hat{i} + 3\hat{j} + 8\hat{k}$ and $\vec{B} = -4\hat{i} + 4\hat{j} + \alpha\hat{k}$

Dot product of these vectors should be equal to zero because they are perpendicular.

$$\therefore \vec{A} \cdot \vec{B} = -8 + 12 + 8\alpha = 0 \Rightarrow 8\alpha = -4 \Rightarrow \alpha = -1/2$$

6. (a) $(\vec{A} + \vec{B})$ is perpendicular to $(\vec{A} - \vec{B})$. Thus

$$(\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = 0$$

$$\text{or } A^2 + \vec{B} \cdot \vec{A} - \vec{A} \cdot \vec{B} - B^2 = 0$$

Because of commutative property of dot product $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

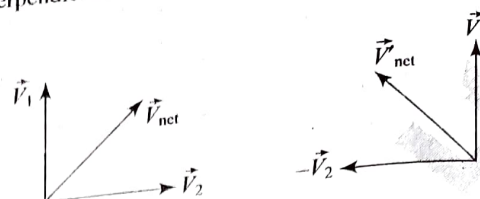
$$\therefore A^2 - B^2 = 0 \text{ or } A = B$$

Thus the ratio of magnitudes $A/B = 1$

7. (c) Force F lie in the x - y plane so a vector along z -axis will be

perpendicular to F .

8. (c)



According to problem $|\vec{V}_1 + \vec{V}_2| = |\vec{V}_1 - \vec{V}_2|$

$$\Rightarrow |\vec{V}_{\text{net}}| = |\vec{V}'_{\text{net}}|$$

So V_1 and V_2 will be mutually perpendicular.

9. (c) $(\hat{i} + \hat{j}) \cdot (\hat{j} + \hat{k}) = 0 + 0 + 1 + 0 = 1$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} = \frac{1}{\sqrt{2} \times \sqrt{2}} = \frac{1}{2} \therefore \theta = 60^\circ$$

10. (a) $\cos \theta = \frac{\vec{P} \cdot \vec{Q}}{PQ} = 1 \therefore \theta = 0^\circ$

11. (c) $P_x = 2 \cos t, P_y = 2 \sin t \therefore \vec{P} = 2 \cos t \hat{i} + 2 \sin t \hat{j}$

$$\vec{F} = \frac{d\vec{P}}{dt} = -2 \sin t \hat{i} + 2 \cos t \hat{j}$$

$$\vec{F} \cdot \vec{P} = 0 \therefore \theta = 90^\circ$$

12. (d) $\vec{AB} = (4\hat{i} + 5\hat{j} + 6\hat{k}) - (3\hat{i} + 4\hat{j} + 5\hat{k}) = \hat{i} + \hat{j} + \hat{k}$
 $\vec{CD} = (4\hat{i} + 6\hat{j}) - (7\hat{i} + 9\hat{j} + 3\hat{k}) = -3\hat{i} - 3\hat{j} - 3\hat{k}$
 $\vec{AB}$ and $\vec{CD}$ are parallel because its cross-products is 0.

13. (c) $\vec{A} \cdot \vec{B} = AB \cos \theta$

In the problem $\vec{A} \cdot \vec{B} = -AB$, i.e., $\cos \theta = -1 \therefore \theta = 180^\circ$
 i.e. $\vec{A}$ and $\vec{B}$ acts in the opposite direction.

14. (a) $\frac{\vec{A} \cdot \vec{B}}{|\vec{i} + \vec{j}|} = \frac{(2\hat{i} + 3\hat{j})(\hat{i} + \hat{j})}{\sqrt{2}} = \frac{2+3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

DPP 3.5

Single Correct Answer Type

1. (c) $\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$

For parallel vectors $\theta = 0^\circ$ or 180° , $\sin \theta = 0$

$$\therefore \vec{A} \times \vec{B} = \hat{0}$$

2. (b) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$

$$= (1 \times 4 - 2 \times -2)\hat{i} + (2 \times 2 - 4 \times 3)\hat{j} + (3 \times -2 - 1 \times 2)\hat{k}$$

$$= 8\hat{i} - 8\hat{j} - 8\hat{k}$$

$\therefore$ Magnitude of

$$\vec{A} \times \vec{B} = |\vec{A} \times \vec{B}| = \sqrt{(8)^2 + (-8)^2 + (-8)^2} = 8\sqrt{3}$$

3. (c) Vector perpendicular to A and B , $\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$

$\therefore$ Unit vector perpendicular to A and B

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A}| \times |\vec{B}| \sin \theta}$$

4. (d) From the property of vector product, we notice that $\vec{C}$ must be perpendicular to the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$ and $(\vec{A} + \vec{B})$ vector also must lie in the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$ must be perpendicular to $(\vec{A} + \vec{B})$ but the cross product $(\vec{A} \times \vec{B})$ gives a vector $\vec{C}$ which cannot be perpendicular to itself. Thus the last statement is wrong.

5. (b) Let $\vec{A} \cdot (\vec{B} \times \vec{A}) = \vec{A} \cdot \vec{C}$

Here $\vec{C} = \vec{B} \times \vec{A}$ Which is perpendicular to both vector

$$\vec{A} \text{ and } \vec{B} \therefore \vec{A} \cdot \vec{C} = 0$$

6. (c) We know that $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$ because the angle between these two is always 90° .

But if the angle between $\vec{A}$ and $\vec{B}$ is 0 or π . Then $\vec{A} \times \vec{B} = \vec{B} \times \vec{A} = 0$.

7. (b) $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ are parallel and opposite to each other. So the angle will be π .

8. (b) Direction of vector A is along z -axis $\therefore \vec{A} = a\hat{k}$

Direction of vector B is towards north $\therefore \vec{B} = b\hat{j}$

$$\text{Now } \vec{A} \times \vec{B} = a\hat{k} \times b\hat{j} = ab(-\hat{j})$$

$\therefore$ The direction is $\vec{A} \times \vec{B}$ is along west.

9. (a) $\vec{A} \times \vec{B}$ is a vector perpendicular to plane $\vec{A} + \vec{B}$ and hence perpendicular to $\vec{A} + \vec{B}$.

10. (d) $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B}) = \vec{A} \times \vec{A} - \vec{A} \times \vec{B} + \vec{B} \times \vec{A} - \vec{B} \times \vec{B}$
 $= 0 - \vec{A} \times \vec{B} + \vec{B} \times \vec{A} - 0 = \vec{B} \times \vec{A} + \vec{B} \times \vec{A} = 2(\vec{B} \times \vec{A})$

$$\begin{aligned} 11. (a) \quad \vec{v} = \vec{\omega} \times \vec{r} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 0 & 4 & -3 \end{vmatrix} \\ &= \hat{i}(6-8) - \hat{j}(-3) + 4\hat{k} - 2\hat{i} + 3\hat{j} + 4\hat{k} \\ |\vec{v}| &= \sqrt{(-2)^2 + (3)^2 + 4^2} = \sqrt{29} \text{ units} \end{aligned}$$

12. (b) Area of parallelogram $= \vec{A} \times \vec{B}$

$$\begin{aligned} &= (\hat{i} + 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k}) \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & -2 & 1 \end{vmatrix} = (8)\hat{i} + (8)\hat{j} - (8)\hat{k} \end{aligned}$$

$$\text{Magnitude} = \sqrt{64 + 64 + 64} = 8\sqrt{3}$$

13. (a) Given $\vec{OA} = \vec{a} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ and $\vec{OB} = \vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$

$$\begin{aligned} \therefore (\vec{a} \times \vec{b}) &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 2 & 1 & -2 \end{vmatrix} \\ &= (12-2)\hat{i} + (4+6)\hat{j} + (3+12)\hat{k} \\ &= 10\hat{i} + 10\hat{j} + 15\hat{k} \Rightarrow |\vec{a} \times \vec{b}| = \sqrt{10^2 + 10^2 + 15^2} \\ &= \sqrt{425} = 5\sqrt{17} \end{aligned}$$

$$\text{Area of } \triangle OAB = \frac{1}{2} |\vec{a} \times \vec{b}| = \frac{5\sqrt{17}}{2} \text{ sq. units sq. unit.}$$

14. (d) $|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A} \cdot \vec{B})$

$$AB \sin \theta = \sqrt{3} AB \cos \theta \Rightarrow \tan \theta = \sqrt{3} \therefore \theta = 60^\circ$$

$$\text{Now } |\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$= \sqrt{A^2 + B^2 + 2AB \left(\frac{1}{2}\right)} = (A^2 + B^2 + AB)^{1/2}$$

DPP 4.1

Single Correct Answer Type

1. (b) $\frac{|\text{Average velocity}|}{|\text{Average speed}|} = \frac{|\text{displacement}|}{|\text{distance}|} \leq 1$
because displacement will either be equal or less than distance. It can never be greater than distance.
2. (c) From given figure, it is clear that the net displacement is zero. So average velocity will be zero.
3. (c) Since displacement is always less than or equal to distance, but never greater than distance. Hence numerical ratio of displacement to the distance covered is always equal to or less than one.
4. (b) Here velocity is decreasing, so acceleration will be opposite to the direction of motion and hence, it will be negative w.r.t the direction of velocity till it comes to rest.
5. (d) Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$
$$= \frac{x}{\frac{2x/5}{v_1} + \frac{3x/5}{v_2}} = \frac{5v_1v_2}{3v_1 + 2v_2}$$
6. (d) Average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{x}{t_1 + t_2}$
$$= \frac{x}{\frac{x/3}{v_1} + \frac{2x/3}{v_2}} = \frac{1}{\frac{1}{3 \times 20} + \frac{2}{3 \times 60}} = 36 \text{ km/hr}$$
7. (d) A man walks from his home to market with a speed of 5 km/h.
Distance = 2.5 km and time = $\frac{d}{v} = \frac{2.5}{5} = \frac{1}{2} \text{ hr}$
and he returns back with speed of 7.5 km/h in rest of time of 10 minutes.
Distance = $7.5 \times \frac{10}{60} = 1.25 \text{ km}$
So, Average speed = $\frac{\text{Total distance}}{\text{Total time}}$
$$= \frac{(2.5 + 1.25) \text{ km}}{(40/60) \text{ hr}} = \frac{45}{8} \text{ km/hr}$$
8. (b) Only direction of displacement and velocity gets changed, acceleration is always directed vertically downward.
9. (c) $v_x = \frac{dx}{dt} = \frac{d}{dt}(3t^2 - 6t) = 6t - 6$. At $t = 1$, $v_x = 0$
 $v_y = \frac{dy}{dt} = \frac{d}{dt}(t^2 - 2t) = 2t - 2$. At $t = 1$, $v_y = 0$
Hence $v = \sqrt{v_x^2 + v_y^2} = 0$

$$10. (b) v = u + \int a dt = u + \int (3t^2 + 2t + 2) dt$$

$$= u + \frac{3t^3}{3} + \frac{2t^2}{2} + 2t = u + t^3 + t^2 + 2t$$

$$= 2 + 8 + 4 + 4 = 18 \text{ m/s} \quad (\text{As } t = 2 \text{ sec})$$

$$11. (b) a = \sqrt{a_x^2 + a_y^2} = \left[\left(\frac{d^2x}{dt^2} \right)^2 + \left(\frac{d^2y}{dt^2} \right)^2 \right]^{\frac{1}{2}}$$

$$\text{Here } \frac{d^2y}{dt^2} = 0. \text{ Hence } a = \frac{d^2x}{dt^2} = 8 \text{ m/s}^2$$

Multiple Correct Answers Type

12. (a, c, d)
Choice (a) is correct, as at the highest point of a straight line vertical motion, velocity is zero but acceleration is not zero. Constant velocity means constant speed as well as same direction throughout. So choice (b) is not correct. If a particle moving with constant speed but its direction is changing the velocity of the particle will change, hence choice (c) is correct. The direction of the velocity of a body can change when a constant force is acting on the particle change, hence choice (d) is correct.
13. (a, b, c)
Initial velocity and acceleration are in opposite directions, so particle stops some where and reverses the direction of motion.
(i) If particle stops before travelling a distance d , then there is only one time the particle will be at distance d from starting point.
(ii) If particle stops at distance d , then two values of time
(iii) If particle stops after a distance d , then three values of time
14. (a, b)
For increasing speed, velocity and acceleration both should be in same direction, either both towards right or both towards left.

Matching Column Type

15. (A $\rightarrow$ r), (B $\rightarrow$ q), (C $\rightarrow$ s), (D $\rightarrow$ p),

We have to analyse slope of each curve i.e., $\frac{dx}{dt}$. For peak points

$\frac{dx}{dt}$ will be zero as x is maximum at peak points.

For graph (A), there is a point (B) for which displacement is zero. So, a matches with (r)

In graph (B), x is positive (> 0) throughout and at point B_1 ,

$v = \frac{dx}{dt} = 0$. Since, at point of curvature changes $a = 0$, So b matches with (q)

In graph (C) slope $v = \frac{dx}{dt}$ is negative, hence velocity will be negative, so matches with (s)

In graph (D), as slope $v = \frac{dx}{dt}$ is positive, hence $v > 0$
Hence, d matches with (p)

DPP 4.2

Subjective Type

1. $\vec{r} = at\hat{i} - bt^2\hat{j}$

$$x = at \quad y = -bt^2$$

$$t = \left(\frac{x}{a}\right) \Rightarrow y = -b\left(\frac{x}{a}\right)^2 \Rightarrow y = \frac{-b}{a^2}x^2.$$

2. $\frac{ds}{dt} = \frac{4}{5}t^2 + 3$

$$\Rightarrow S_1 = \int_0^5 ds = \int_0^5 \left(\frac{4}{5}t^2 + 3\right) dt$$

$$S_1 = \left[\frac{4}{15}t^3 + 3t\right]_0^5 = \frac{145}{3} \text{ m}$$

$$S_2 = \int_0^{10} ds = \int_5^{10} (3t + 8) dt$$

$$S_2 = \left[\frac{3t^2}{2} + 8t\right]_5^{10} = \frac{305}{2} \text{ m}$$

$$S = S_1 + S_2 = \left(\frac{145}{3} + \frac{305}{2}\right) \text{ m}$$

Single Correct Answer Type

3. (c) $\frac{dv}{dt} = bt \Rightarrow dv = bt dt \Rightarrow v = \frac{bt^2}{2} + K_1$

At $t = 0, v = v_0 \Rightarrow K_1 = v_0$

We get $v = \frac{1}{2}bt^2 + v_0$

Again $\frac{dx}{dt} = \frac{1}{2}bt^2 + v_0 \Rightarrow x = \frac{1}{2} \cdot \frac{bt^3}{3} + v_0t + K_2$

At $t = 0, x = 0 \Rightarrow K_2 = 0$

$$\therefore x = \frac{1}{6}bt^3 + v_0t$$

4. (b) $v = 4t^3 - 2t$ (given) $\therefore a = \frac{dv}{dt} = 12t^2 - 2$

and $x = \int_0^t v dt = \int_0^t (4t^3 - 2t) dt = t^4 - t^2$

When particle is at 2 m from the origin $t^4 - t^2 = 2$

$$\Rightarrow t^4 - t^2 - 2 = 0 \quad (t^2 - 2)(t^2 + 1) = 0 \Rightarrow t = \sqrt{2} \text{ sec}$$

Acceleration at $t = \sqrt{2} \text{ sec}$ given by,

$$a = 12t^2 - 2 = 12 \times 2 - 2 = 22 \text{ m/s}^2$$

5. (a) $\frac{dt}{dx} = 2\alpha x + \beta \Rightarrow v = \frac{1}{2\alpha x + \beta}$

$$\therefore a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$$

$$a = v \frac{dv}{dx} = \frac{-v \cdot 2\alpha}{(2\alpha x + \beta)^2} = -2\alpha \cdot v \cdot v^2 = -2\alpha v^3$$

$$\therefore \text{Retardation} = 2\alpha v^3$$

6. (d) $3t = \sqrt{3x} + 6 \Rightarrow 3x = (3t - 6)^2$

$$\Rightarrow x = 3t^2 - 12t + 12$$

$$v = \frac{dx}{dt} = 6t - 12, \text{ for } v = 0, t = 2 \text{ sec}$$

$$x = 3(2)^2 - 12 \times 2 + 12 = 0$$

7. (c) $v = (180 - 16x)^{1/2}$

As $a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$

$$\therefore a = \frac{1}{2}(180 - 16x)^{-1/2} \times (-16) \left(\frac{dx}{dt}\right)$$

$$= -8(180 - 16x)^{-1/2} \times v$$

$$= -8(180 - 16x)^{-1/2} \times (180 - 16x)^{1/2} = -8 \text{ m/s}^2$$

8. (b) $x = 4(t - 2) + a(t - 2)^2$

At $t = 0, x = -8 + 4a = 4a - 8$

$$v = \frac{dx}{dt} = 4 + 2a(t - 2)$$

At $t = 0, v = 4 - 4a = 4(1 - a)$

But acceleration, $a = \frac{d^2x}{dt^2} = 2a$

9. (a) $\vec{r} = (t^2 - 4t + 6)\hat{i} + t^2\hat{j}; \vec{v} = \frac{d\vec{r}}{dt}$

$$= (2t - 4)\hat{i} + 2t\hat{j}, \vec{a} = \frac{d\vec{v}}{dt} = 2\hat{i} + 2\hat{j}$$

if velocity vector and acceleration vector are perpendicular

$$\vec{a} \cdot \vec{v} = 0$$

$$(2\hat{i} + 2\hat{j}) \cdot ((2t - 4)\hat{i} + 2t\hat{j}) = 0$$

$$8t - 8 = 0, \quad t = 1 \text{ sec}$$

Multiple Correct Answers Type

10. (a, b, c)

On the curve $y = x^2$ at $x = 1/2 \Rightarrow y = \frac{1}{4}$

Hence the coordinate $\left(\frac{1}{2}, \frac{1}{4}\right)$

Differentiating: $y = x^2, v_y = 2xv_x$

$$v_y = 2\left(\frac{1}{2}\right)(4) = 4 \text{ m/s}$$

Which satisfies the line

$$4x - 4y - 1 = 0 \quad (\text{tangent to the curve})$$

And magnitude of velocity:

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = 4\sqrt{2} \text{ m/s}$$

11. (a, d)

Given $v = 10\sqrt{x}$

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Putting $x = 0$ gives $v = 0$, so initial velocity of the particle is zero.

$$a = \frac{dv}{dt} = \frac{10}{2\sqrt{x}} \frac{dx}{dt}$$

$$\Rightarrow a = \frac{5}{\sqrt{x}} v = \frac{5}{\sqrt{x}} 10\sqrt{x} = 50 \text{ m/s}^2$$

12. (a, b, d)

$$x = 50t - 5t^2$$

$$v = \frac{dx}{dt} = 50 - 10t$$

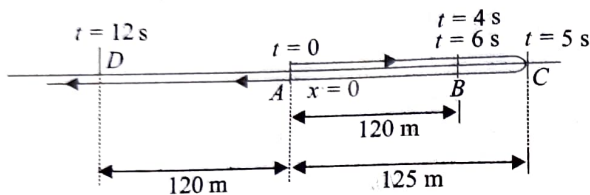
Put $v = 0 \Rightarrow t = 5 \text{ s}$

$$\text{at } t = 5 \text{ s: } x_1 = 50 \times 5 - 5(5)^2 = 125 \text{ m}$$

$$\text{at } t = 4 \text{ s: } x_2 = 50 \times 4 - 5(4)^2 = 120 \text{ m}$$

$$\text{at } t = 6 \text{ s: } x_3 = 50 \times 6 - 5(6)^2 = 120 \text{ m}$$

$$\text{at } t = 12 \text{ s: } x_4 = 50 \times 12 - 5(12)^2 = -120 \text{ m}$$



Distance travelled in 6 s:

$$= AC + CB = 125 + 5 = 130 \text{ m}$$

Velocity at no two points has same magnitude and direction.

Finally particle will continue to move towards left and its distance from origin will go on increasing.

DPP 4.3

Subjective Type

1. $u = +8 \text{ m/s}$, $a = -4 \text{ m/s}^2$

$$v = 0$$

$$\Rightarrow 0 = 8 - 4t \text{ or } t = 2 \text{ sec}$$

Displacement in first 2 sec

$$S_1 = 8 \times 2 + \frac{1}{2} \cdot (-4) \cdot 2^2 = 8 \text{ m}$$

Displacement in next 3 sec

$$S_2 = 0 \times 3 + \frac{1}{2} \cdot (-4) \cdot 3^2 = -18 \text{ m}$$

$$\text{Distance travelled} = |S_1| + |S_2| = 26 \text{ m}$$

2. Let ' t ' second be the reaction time of the driver to be able to put brakes and let ' a ' be the acceleration of the car, when full brakes are applied.

(i) Distance travelled by the car, after the driver has seen the signal and before the driver has just applied the brakes (i.e. due to reaction time)

$$= \text{velocity} \times t$$

$$= 72 \text{ km/hr} \times t = 20 t \text{ m/sec}$$

Thus the brakes stopped the car within the distance of $(30 - 20t) \text{ m}$.

$$a = \frac{0^2 - 20^2}{2 \times (30 - 20t)} = \frac{-20^2}{2 \times (30 - 20t)} = -\frac{20}{30t}$$

Similarly

$$a = \frac{(0 - 10^2)}{2(10 - 10t)} = \frac{-5}{1-t}; \quad a = \frac{(0 - 15^2)}{2(x - 15t)}$$

when x : distance travelled by car in third case

$$\frac{20}{3-2t} = \frac{5}{1-t} = \frac{15^2}{2(x-15t)}$$

$$t = \frac{1}{2} \text{ sec.} \quad x = 18.75 \text{ m}$$

Single Correct Answer Type

3. (b) As $S = ut + \frac{1}{2}at^2 \therefore S_1 = \frac{1}{2}a(10)^2 = 50a$ (i)

$$\text{As } v = u + at$$

$$\therefore \text{Velocity acquired by particle in 10 sec } v = a \times 10$$

$$\text{For next 10 sec, } S_2 = (10a) \times 10 + \frac{1}{2}(a) \times (10)^2$$

$$S_2 = 150a \quad \text{(ii)}$$

$$\text{From (i) and (ii) } S_1 = S_2/3$$

4. (a) From $S = ut + \frac{1}{2}at^2$

$$S_1 = \frac{1}{2}a(P-1)^2 \text{ and } S_2 = \frac{1}{2}aP^2 \quad [\text{As } u = 0]$$

$$\text{From } S_n = u + \frac{a}{2}(2n-1)$$

$$S_{(P^2-P+1)\text{th}} = \frac{a}{2}[2(P^2-P+1)-1] = \frac{a}{2}[2P^2-2P+1]$$

$$\text{It is clear that } S_{(P^2-P+1)\text{th}} = S_1 + S_2$$

5. (a) Let initial ($t = 0$) velocity of particle = u

For first 5 sec motion $s_5 = 10 \text{ metre}$

$$s = ut + \frac{1}{2}at^2 \Rightarrow 10 = 5u + \frac{1}{2}a(5)^2$$

$$2u + 5a = 4 \quad \text{(i)}$$

For first 8 sec of motion $s_8 = 20 \text{ metre}$

$$20 = 8u + \frac{1}{2}a(8)^2 \Rightarrow 2u + 8a = 5 \quad \text{(ii)}$$

$$\text{By solving } u = \frac{7}{6} \text{ m/s and } a = \frac{1}{3} \text{ m/s}^2$$

Now distance travelled by particle in total 10 sec

$$s_{10} = u \times 10 + \frac{1}{2}a(10)^2$$

By substituting the value of u and a we will get $s_{10} = 28.3 \text{ m}$

so the distance in last 2 sec = $s_{10} - s_8$

$$= 28.3 - 20 = 8.3 \text{ m}$$

6. (a) Velocity acquired by body in 10 sec

$$v = 0 + 2 \times 10 = 20 \text{ m/s}$$

and distance travelled by it in 10 sec

$$S_1 = \frac{1}{2} \times 2 \times (10)^2 = 100 \text{ m}$$

then it moves with constant velocity (20 m/s) for 30 sec

$$S_2 = 20 \times 30 = 600 \text{ m}$$

After that due to retardation (4 m/s^2) it stops

$$S_3 = \frac{v^2}{2a} = \frac{(20)^2}{2 \times 4} = 50 \text{ m}$$

Total distance travelled $S_1 + S_2 + S_3 = 750 \text{ m}$

For first part,

7. (c) $u = 0$, $t = T$ and acceleration $= a$

$$\therefore v = 0 + aT = aT \text{ and } S_1 = 0 + \frac{1}{2} aT^2 = \frac{1}{2} aT^2$$

For second part,

$u = aT$, retardation $= a_1$, $v = 0$ and time taken $= T_1$ (let)

$$\therefore 0 = u - a_1 T_1 \Rightarrow aT = a_1 T_1$$

$$\text{and from } v^2 = u^2 - 2aS_2 \Rightarrow S_2 = \frac{u^2}{2a_1} = \frac{1}{2} \frac{a^2 T^2}{a_1}$$

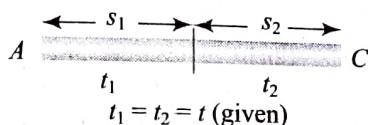
$$S_2 = \frac{1}{2} aT \times T_1 \quad \left(\text{As } a_1 = \frac{aT}{T_1} \right)$$

$$\therefore v_{av} = \frac{S_1 + S_2}{T + T_1} = \frac{\frac{1}{2} aT^2 + \frac{1}{2} aT \times T_1}{T + T_1}$$

$$= \frac{\frac{1}{2} aT (T + T_1)}{T + T_1} = \frac{1}{2} aT$$

$$8. (c) \therefore S_1 = ut + \frac{1}{2} at^2$$

and velocity after first t sec



$$v = u + at$$

$$\text{Now, } S_2 = vt + \frac{1}{2} at^2$$

$$= (u + at)t + \frac{1}{2} at^2$$

$$\text{Equation (ii) - (i)} \Rightarrow S_2 - S_1 = at^2$$

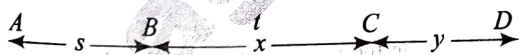
$$\Rightarrow a = \frac{S_2 - S_1}{t^2} = \frac{65 - 40}{(5)^2} = 1 \text{ m/s}^2$$

From equation (i), we get,

$$S_1 = ut + \frac{1}{2} at^2 \Rightarrow 40 = 5u + \frac{1}{2} \times 1 \times 25$$

$$\Rightarrow 5u = 27.5 \therefore u = 5.5 \text{ m/s}$$

9. (c) Let car starts from point A from rest and moves up to point B with acceleration f



$$\text{Velocity of car at point B, } v = \sqrt{2fs} \quad [\text{As } v^2 = u^2 + 2as]$$

Car moves distance BC with this constant velocity in time t

$$x = \sqrt{2fs} \cdot t \quad (i) \quad [\text{As } s = ut]$$

So the velocity of car at point C also will be $\sqrt{2fs}$ and finally car stops after covering distance y .

$$\text{Distance CD} \Rightarrow y = \frac{(\sqrt{2fs})^2}{2(f/2)} = \frac{2fs}{f} = 2S \quad (ii)$$

$$[\text{As } v^2 = u^2 - 2as \Rightarrow s = u^2/2a]$$

So, the total distance $AD = AB + BC + CD = 15S$ (given)

$$\Rightarrow S + x + 2S = 15S \Rightarrow x = 12S$$

Substituting the value of x in equation (i) we get

$$x = \sqrt{2fs} \cdot t \Rightarrow 12S = \sqrt{2fs} \cdot t \Rightarrow 144S^2 = 2fs \cdot t^2$$

$$\Rightarrow S = \frac{1}{72} \text{ ft}^2$$

10. (c) Let man will catch the bus after ' t ' sec. So he will cover distance ut .

Similarly distance travelled by the bus will be $\frac{1}{2} at^2$. For the given condition

$$ut = 45 + \frac{1}{2} at^2 = 45 + 1/25 t^2 \quad [\text{As } a = 2.5 \text{ m/s}^2]$$

$$\Rightarrow u = \frac{45}{t} + 1.25t$$

To find the minimum value of u

$$\frac{du}{dt} = 0 \text{ so we get } t = 6 \text{ sec, then}$$

$$u = \frac{45}{6} + 1.25 \times 6 = 7.5 + 7.5 = 15 \text{ m/s}$$

11. (b) Let car B catches, car A after ' t ' sec, then

$$60t + 2.5 = 70t - \frac{1}{2} \times 20 \times t^2$$

$$\Rightarrow 10t^2 - 10t + 2.5 = 0 \Rightarrow t^2 - t + 0.25 = 0$$

$$\therefore t = \frac{1 \pm \sqrt{1 - 4 \times (0.25)}}{2} = \frac{1}{2} \text{ hr}$$

12. (d) As $v^2 = u^2 + 2as \Rightarrow (2u)^2 = u^2 + 2as \Rightarrow 2as = 3u^2$

Now, after covering an additional distance s , if velocity becomes v , then

$$v^2 = u^2 + 2a(2s) = u^2 + 4as = u^2 + 6u^2 = 7u^2$$

$$\therefore v = \sqrt{7}u$$

DPP 4.4

Single Correct Answer Type

1. (d) Let the body after time $t/2$ be at x from the top, then

$$x = \frac{1}{2} g \frac{t^2}{4} = \frac{gt^2}{8} \quad (i)$$

$$h = \frac{1}{2} gt^2 \quad (ii)$$

Eliminate t from (i) and (ii), we get $x = \frac{h}{4}$

$$\therefore \text{Height of the body from the ground} = h - \frac{h}{4} = \frac{3h}{4}$$

2. (c) Since direction of v is opposite to the direction of g and h so from equation of motion

$$h = -vt + \frac{1}{2} gt^2$$

$$\Rightarrow gt^2 - 2vt - 2h = 0$$

$$\Rightarrow t = \frac{2v \pm \sqrt{4v^2 + 8gh}}{2g}$$

$$\Rightarrow t = \frac{v}{g} \left[1 + \sqrt{1 + \frac{2gh}{v^2}} \right]$$

3. (b) Average velocity = 0 because net displacement of the body is zero.

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Time of flight}} = \frac{2H_{\max}}{2u/g}$$

$$\Rightarrow v_{av} = \frac{2u^2/2g}{2u/g} \Rightarrow v_{av} = u/2$$

Velocity of projection = v (given)

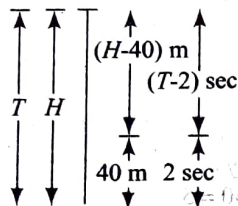
$$\therefore v_{av} = v/2$$

4. (b) For vertically upward motion, $h_1 = v_0 t - \frac{1}{2} g t^2$ and for vertically

$$\text{downward motion, } h_2 = v_0 t + \frac{1}{2} g t^2$$

$$\therefore \text{Total distance covered in } t \text{ sec } h = h_1 + h_2 = 2v_0 t$$

5. (b) Let height of minaret is H and body take time T to fall from top to bottom.



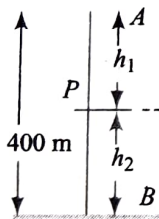
$$H = \frac{1}{2} g T^2 \quad (i)$$

In last 2 sec, body travels distance of 40 meter so in $(T-2)$ sec distance travelled = $(H-40)$ m.

$$(H-40) = \frac{1}{2} g (T-2)^2 \quad (ii)$$

By solving (i) and (ii) $T = 3$ sec and $H = 45$ m

6. (c) Let both balls meet at point P after time t .



The distance travelled by ball A, $h_1 = \frac{1}{2} g t^2$

The distance travelled by ball B, $h_2 = ut - \frac{1}{2} g t^2$

$$h_1 + h_2 = 400 \text{ m} \Rightarrow ut = 400, t = 400/50 = 8 \text{ sec}$$

$$\therefore h_1 = 320 \text{ m and } h_2 = 80 \text{ m}$$

7. (b) The distance traveled in last second.

$$S_{\text{Last}} = u + \frac{g}{2} (2t-1) = \frac{1}{2} \times 10(2t-1) = 5(2t-1)$$

and distance traveled in first three second,

$$S_{\text{Three}} = 0 + \frac{1}{2} \times 10 \times 9 = 45 \text{ m}$$

According to problem $S_{\text{Last}} = S_{\text{Three}}$

$$\Rightarrow 5(2t-1) = 45 \Rightarrow 2t-1 = 9 \Rightarrow t = 5 \text{ sec}$$

8. (b) At maximum height velocity $v = 0$

We know that $v = u + at$, hence

$$0 = u - gT \Rightarrow u = gT$$

When $v = \frac{u}{2}$, then

$$\frac{u}{2} = u - gt \Rightarrow gt = \frac{u}{2} \Rightarrow gt = \frac{gT}{2} \Rightarrow t = \frac{T}{2}$$

Hence at $t = \frac{T}{2}$, it acquires velocity $\frac{u}{2}$

9. (b) Let at point A initial velocity of body is equal to zero

$$\text{for path AB: } v^2 = 0 + 2gh \quad (i)$$

$$\text{for path AC: } (2v)^2 = 0 + 2gx \quad (ii)$$

$$4v^2 = 2gx$$

Solving (i) and (ii) $x = 4h$

10. (d) Interval of ball throw = 2 sec.

If we want that minimum three (more than two)

balls remain in air then time of flight of first ball must be greater than 4 sec.

$$T > 4 \text{ sec}$$

$$\frac{2u}{g} > 4 \text{ sec} \Rightarrow u > 20 \text{ m/s}$$

for $u = 20$ m/s. First ball will just strike the ground (in sky)

Second ball will be at highest point (in sky)

Third ball will be at point of projection or at ground (not in sky)

Multiple Correct Answers Type

11. (a, b, c)

At $t = \frac{T}{4}$ and $t = \frac{3T}{4}$, the stone is at same height.

Hence average velocity in this time interval is zero.

Change in velocity in same time interval is same for a particle moving with constant acceleration.

Let H be maximum height attained by stone, then distance travelled from $t = 0$ to $t = \frac{T}{4}$ is $\frac{3}{4}H$ and from $t = \frac{T}{4}$ to

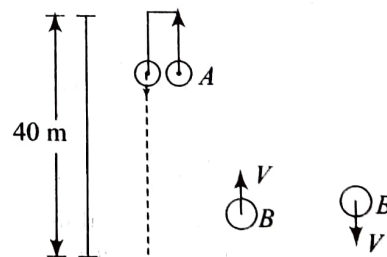
$t = \frac{3T}{4}$ distance travelled is $\frac{H}{4}$.

From $t = \frac{T}{2}$ to $t = T$ sec distance travelled is H and from

$t = \frac{T}{2}$ to $t = \frac{3T}{4}$ distance travelled is $\frac{H}{4}$.

12. (a, c, d)

For Ball A



$$S = -40 \text{ m; } u = +10 \text{ m/s;}$$

$$a = -10 \text{ m/s}^2$$

$$S = ut + \frac{1}{2} at^2 - 40 = 10t + \frac{1}{2} (-10)t^2$$

$$\Rightarrow t = -2, 4 \Rightarrow t = 4 \text{ sec}$$

(Ball A will reach ground after 4 sec)

In two seconds, displacement of Ball A is

$$S(2) = 10t - \frac{1}{2}(10)t^2 = 10(2) - \frac{4(10)}{2} = 0$$

i.e. the ball B is thrown when the first ball have come back to the initial position, having velocity 10 m/s in opposite direction (downward direction).

For both the balls to reach ground simultaneously initial velocity in y-direction must be same for both the balls.

Thus the second ball was projected vertically downwards with speed of 10 m/s.

Distance travelled by first ball (a) in two seconds

$$= 2 \times [10 \times 1 - \frac{1}{2} \times 10 \times 1^2] = 2 \times 5 = 10 \text{ m.}$$

After that ball A and Ball B will travel same distance (= 40 m).

Since both balls have same velocity and acceleration at $t = 2$ sec, hence they will hit the ground with same velocity.

13. (a, c)

Height of balloon after 8 s:

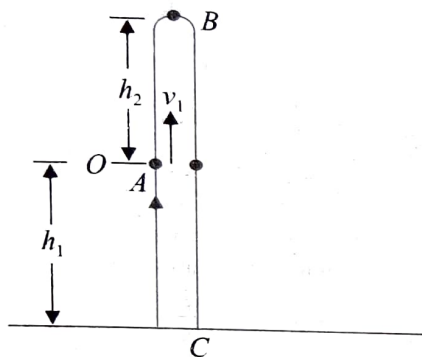
$$h_1 = \frac{1}{2} \times (1.25) \times (8)^2 = 40 \text{ m}$$

Velocity of balloon at this time:

$$v_1 = 1.25 \times 8 = 10 \text{ m/s}$$

Further height raised by stone:

$$h_2 = \frac{v_1^2}{2g} = \frac{10^2}{2 \times 10} = 5 \text{ m}$$



Distance covered by stone after it is released:

$$= AB + BC = 5 + 45 = 50 \text{ m}$$

and displacement = $AC = 40 \text{ m}$

for stone: from A to C:

$$-h_1 = v_1 t - \frac{1}{2} g t^2 \Rightarrow -40 = 10t - 5t^2$$

$$\Rightarrow t^2 - 2t - 8 = 0 \Rightarrow (t - 4)(t + 2) = 0$$

$$\Rightarrow t = 4 \text{ s}$$

Comprehension Type

14. (a) Let the speeds of the two balls (1 and 2) be v_1 and v_2 where

$$v_1 = 2v, v_2 = v$$

If y_1 and y_2 and the distance covered by the balls 1 and 2, respectively, before coming to rest, then

$$y_1 = \frac{v_1^2}{2g} = \frac{4v^2}{2g} \quad \text{and} \quad y_2 = \frac{v_2^2}{2g} = \frac{v^2}{2g}$$

$$\text{Since } y_1 - y_2 = 15 \text{ m, } \frac{4v^2}{2g} - \frac{v^2}{2g}$$

$$\frac{4v^2}{2g} - \frac{v^2}{2g} = 15$$

$$\Rightarrow v = \sqrt{5 \times (2 \times 10)} = 10 \text{ m/s}$$

Clearly, $v_1 = 20 \text{ m/s}$ and $v_2 = 10 \text{ m/s}$

$$15. (d) \text{ As } y_1 = \frac{v_1^2}{2g} = \frac{(20)^2}{2 \times 10} = 20 \text{ m}$$

$$y_2 = y_1 - 15 \text{ m} = 5 \text{ m}$$

If t_2 is the time taken by the ball 2 to cover

a distance of 5 m, then from $y_2 = v_2 t - \frac{1}{2} g t_2^2$

$$5 = 10t_2 - 5t_2^2 \quad \text{or} \quad t_2^2 - 2t_2 + 1 = 0$$

Where $\Rightarrow t_2 = 1 \text{ sec}$

Since t_1 (time taken by ball 1 to cover distance of 20 m) is 2 sec, time interval between the two throws

$$= t_1 - t_2 = 2 \text{ s} - 1 \text{ s} = 1 \text{ s}$$

DPP 4.5

Single Correct Answer Type

1. (d) Total distance = $130 + 120 = 250 \text{ m}$

Relative velocity = $30 - (-20) = 50 \text{ m/s}$

Hence $t = 250/50 = 5 \text{ s}$

2. (b) Relative velocity of bird w.r.t train = $25 + 5 = 30 \text{ m/s}$

time taken by the bird to cross the train $t = \frac{210}{30} = 7 \text{ sec}$

3. (a) Effective speed of the bullet

= speed of bullet + speed of police jeep

$$= 180 \text{ m/s} + 45 \text{ km/h} = (180 + 12.5) \text{ m/s} = 192.5 \text{ m/s}$$

Speed of thief's jeep = $153 \text{ km/h} = 42.5 \text{ m/s}$

Velocity of bullet w.r.t thief's car = $192.5 - 42.5 = 150 \text{ m/s}$

4. (a) When two particles moves towards each other then

$$v_1 + v_2 = 6$$

When these particles moves in the same direction then

$$v_1 - v_2 = 4$$

By solving $v_1 = 5$ and $v_2 = 1 \text{ m/s}$

5. (d) For the round trip he should cross perpendicular to the river

$$\therefore \text{Time for trip to that side} = \frac{1 \text{ km}}{4 \text{ km/hr}} = 0.25 \text{ hr}$$

To come back, again he takes 0.25 hr to cross the river.

Total time is 30 min, he goes to the other bank and comes back to the same point.

6. (a) As the trains are moving in the same direction. So the relative speed ($v_1 - v_2$) and by applying retardation final relative speed becomes zero.

$$\text{From } v = u - at \Rightarrow 0 = (v_1 - v_2) - at \Rightarrow t = \frac{v_1 - v_2}{a}$$

7. (d) The relative velocity of policeman w.r.t. thief = $10 - 9 = 1 \text{ m/s}$

$$\therefore \text{Time taken by police to catch the thief} = \frac{100}{1} = 100 \text{ sec}$$

8. (b) Relative velocity of one train w.r.t. other

$$= 10 + 10 = 20 \text{ m/s.}$$

Relative acceleration = $0.3 + 0.2 = 0.5 \text{ m/s}^2$

If trains cross each other then from $s = ut + \frac{1}{2} at^2$

$$\text{As, } s = s_1 + s_2 = 100 + 125 = 225$$

$$\Rightarrow 225 = 20t + \frac{1}{2} \times 0.5 \times t^2 \Rightarrow 0.5t^2 + 40t - 450 = 0$$

$$\Rightarrow t = -\frac{40 \pm \sqrt{1600 + 4 \cdot (0.005) \times 450}}{1} = -40 \pm 50$$

$\therefore t = 10$ sec (Taking +ve value).

9. (b) The two car (say A and B) are moving with same velocity, the relative velocity of one (say B) with respect to the other A, $A, \vec{v}_{BA} = \vec{v}_B - \vec{v}_A = v - v = 0$

So the relative separation between them (= 5 km) always remains the same.

Now if the velocity of car (say C) moving in opposite direction to A and B, is $\vec{v}_C$ relative to ground then the velocity of car C relative to A and B will be $\vec{v}_{rel} = \vec{v}_C - \vec{v}$

But as $\vec{v}$ is opposite to v_C

So $v_{rel} = v_C - (-30) = (v_C + 30)$ km/hr.

So, the time taken by it to cross the cars A and B

$$t = \frac{d}{v_{rel}} \Rightarrow \frac{4}{60} = \frac{5}{v_C + 30}$$

$$\Rightarrow v_C = 45 \text{ km/hr.}$$

10. (b) Boat covers distance of 16 km in a still water in 2 hours.

$$\text{i.e. } v_B = \frac{16}{2} = 8 \text{ km/hr}$$

Now velocity of water $\Rightarrow v_w = 4$ km/hr.

Time taken for going upstream

$$t_1 = \frac{8}{v_B - v_w} = \frac{8}{8 - 4} = 2 \text{ hr}$$

(As water current oppose the motion of boat)

Time taken for going down stream

$$t_2 = \frac{8}{v_B + v_w} = \frac{8}{8 + 4} = \frac{8}{12} \text{ hr}$$

(As water current helps the motion of boat)

$$\therefore \text{Total time} = t_1 + t_2 = \left(2 + \frac{8}{12}\right) \text{ hr or } 2 \text{ hr } 40 \text{ min}$$

Multiple Correct Answers Type

11. (a, c, d)

The acceleration of the body separated from another accelerated body is independent of the acceleration of parent body

Thus acceleration of ball is equal to g

For first case (when lift is ascending)

$$t_1 = \frac{2v}{g + a}$$

for second case (when lift is descending)

$$t_2 = \frac{2v}{g - a}$$

on solving equation (i) and (ii) we get

$$V = \frac{gt_1t_2}{t_1 + t_2}; \quad a = \frac{g(t_2 - t_1)}{t_1 + t_2}$$

Comprehension Type

12. (a) Subscript B stands for bullet, E for elevator, and G for ground.

$$\vec{V}_{BE} = 15 \text{ m/s } (\uparrow)$$

$$\vec{a}_{BE} = \vec{a}_{BG} - \vec{a}_{EG}$$

$$= -g - (-a) = (g + a)(\downarrow)$$

In the frame of elevator (for bullet)

$$s = ut + \frac{1}{2}at^2$$

$$-1.5 = 15t - \frac{1}{2}(g + a)t^2 \text{ given } t = 2 \text{ sec}$$

$$-1.5 = 30 - 2(g + a)$$

$$\vec{a} = \vec{a}_{EG} = 15.75 - g = 5.75 \text{ m/s}^2 (\uparrow)$$

13. (d) Displacement of floor is elevator in 2 seconds relative to ground

$$= 10t + \frac{1}{2}(5.75)t^2$$

$$= 20 + 11.50 = 31.50 \text{ m}$$

The bullet was projected 1.5 m above the floor and it returns back to the floor in 2 seconds.

So, displacement of bullet relative to ground = $31.50 - 1.5 = 30.0 \text{ m}$

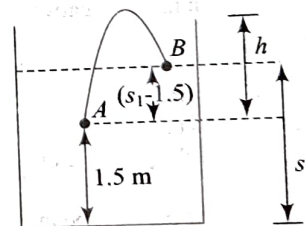
$$14. (c) \vec{V}_{BG} = \vec{V}_{BE} + \vec{V}_{EG}$$

$$= 15 + 10 = 25 \text{ m/s}$$

$$h = \frac{V_{BG}^2}{2a_{BG}} = \frac{(25)^2}{2 \times 10} = 31.25 \text{ m}$$

Required height = $51.5 + 31.25 = 82.75 \text{ m}$

15. (a) The bullet is projected from A but it lands at B because the floor of the elevator has moved up in 2 seconds by distance



$$S_1 = 10t + \frac{1}{2}(5.75)t^2 = 31.50 \text{ m}$$

Distance travelled by bullet

$$= 2h - (S_1 - 1.5)$$

$$= 2h - S_1 + 1.5$$

$$= 2(31.25) - 31.50 + 1.5 = 32.0 \text{ m}$$

DPP 4.6

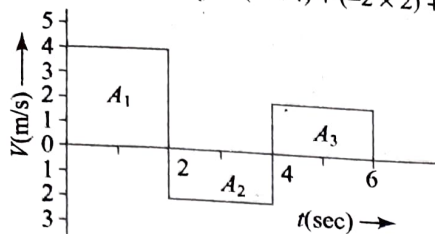
Single Correct Answer Type

1. (d) Up to time t_1 slope of the graph is constant and after t_1 slope is zero i.e. the body travels with constant speed up to time t_1 and then stops.

2. (c) $v^2 = u^2 + 2as$. If $u = 0$ then $v^2 \propto s$

i.e., graph should be parabola symmetric to displacement axis.

3. (a) Displacement = Summation of all the area with sign
 $= (A_1) + (-A_2) + (A_3) = (2 \times 4) + (-2 \times 2) + (2 \times 2)$



$\therefore$ Displacement = 8 m

Distance = Summation of all the areas without sign

$= |A_1| + |-A_2| + |A_3| = |8| + |-4| + |4| = 8 + 4 + 4$

$\therefore$ Distance = 16 m

4. (c) From given $a-t$ graph it is clear that acceleration is increasing at constant rate

$\therefore \frac{da}{dt} = k$ (constant) $\Rightarrow a = kt$ (by integration)

$\Rightarrow \frac{dv}{dt} = kt \Rightarrow dv = ktdt$

$\Rightarrow \int dv = k \int t dt \Rightarrow v = \frac{kt^2}{2}$

i.e., v is dependent on time parabolically and parabola is symmetric about v -axis.

and suddenly acceleration becomes zero, i.e., velocity becomes constant.

Hence (c) is most probable graph.

5. (d) In the positive region the velocity decreases linearly (during rise) and in the negative region velocity increases linearly (during fall) and the direction is opposite to each other during rise and fall, hence fall is shown in the negative region.

6. (c) For upward motion

Effective acceleration = $-(g + a)$

and for downward motion

Effective acceleration = $(g - a)$

But both are constants. So the slope of speed-time graph will be constant.

7. (c) At time t_1 the velocity of ball will be maximum and it goes on decreasing with respect to time.

At the highest point of path its velocity becomes zero, then it increases but direction is reversed

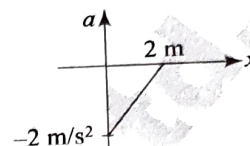
This explanation match with graph (c).

8. (c) From the graph, we can write

$v = -x + 2$

$a = \frac{dv}{dt} = -\frac{dx}{dt} = -v$

$= -(-x + 2) = x - 2$



Multiple Correct Answers Type

9. (a, b, d)

$OP < OQ$, so A lives closer.

A starts at $t = 0$, whereas B starts at $t = t_1$. So A starts earlier than B. Their average velocity is not equal because their displacement OP and OR are not same.

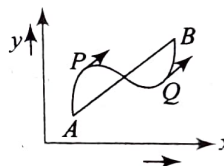
B overtakes A, where both the graphs intersect.

10. (a, c, d)

The particle cannot have zero acceleration along path 1 as direction of velocity is changing with time.

Average velocity = $\frac{\text{Total Displacement}}{\text{Total time taken}} = \frac{AB}{t}$

Hence average velocity is equal along both paths.



Average velocity is along AB, instantaneous at P and Q is in same direction.

Matching Column Type

11. (A $\rightarrow$ r), (B $\rightarrow$ q), (C $\rightarrow$ s), (D $\rightarrow$ p)

- a is +ve constant $\Rightarrow$ velocity will increase linearly with time and position will increase parabolically with time.
- a is -ve constant $\Rightarrow$ velocity will decrease linearly with time and position will decrease parabolically with time.
- First acceleration is +ve and then -ve $\Rightarrow$ velocity will first increase and then decrease. Also, position of the particle will increase parabolically.
- First acceleration is -ve and then +ve $\Rightarrow$ velocity will decrease linearly and then increase.

CHAPTER 5

DPP 5.1

Single Correct Answer Type

1. (c) Instantaneous velocity of rising mass after t sec will be

$$v_t = \sqrt{v_x^2 + v_y^2}$$

where $v_x = v \cos \theta$ = Horizontal component of velocity

$v_y = v \sin \theta - gt$ = Vertical component of velocity

$$v_t = \sqrt{(v \cos \theta)^2 + (v \sin \theta - gt)^2}$$

$$v_t = \sqrt{v^2 + g^2 t^2 - 2v \sin \theta \cdot gt}$$

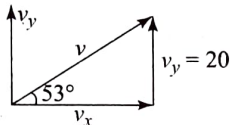
2. (b)

Two second before maximum height

$$0 = v_y - gt$$

$$v_y = gt \Rightarrow v_y = g \times 2 = 20 \text{ m/s}$$

$$\tan 53^\circ = \frac{20}{v_x} \quad v_x = 15 \text{ m/s}$$



at maximum height $v = v_x = 15 \text{ m/s}$

3. (d) $R = \frac{u^2 \sin 2\theta}{g} = \frac{2u_x u_y}{g}$

$\therefore$ Range $\propto$ horizontal initial velocity (u_x)

In path 4 range is maximum so football possesses maximum horizontal velocity in this path.

4. (a) When the angle of projection is very far from 45° , then range will be minimum.

5. (a) If air resistance is taken into consideration, then range and maximum height, both will decrease

6. (a) $H_1 = \frac{u^2 \sin^2 \theta}{2g}$ and $H_2 = \frac{u^2 \sin^2 (90 - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$

$$H_1 H_2 = \frac{u^2 \sin^2 \theta}{2g} \times \frac{u^2 \cos^2 \theta}{2g} = \frac{(u^2 \sin \theta \cos \theta)^2}{16g^2} = \frac{R^2}{16}$$

$$\therefore R = 4\sqrt{H_1 H_2}$$

7. (d) Standard equation of projectile motion

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

Comparing with given equation

$$A = \tan \theta \text{ and } B = \frac{g}{2u^2 \cos^2 \theta}$$

$$\text{So } \frac{A}{B} = \frac{\tan \theta \times 2u^2 \cos^2 \theta}{g} = 40$$

(As $\theta = 45^\circ$, $u = 20 \text{ m/s}$, $g = 10 \text{ m/s}^2$)

8. (b) For same range angle of projection should be θ and $90 - \theta$

$$\text{So, time of flights } t_1 = \frac{2u \sin \theta}{g} \text{ and}$$

$$t_2 = \frac{2u \sin (90 - \theta)}{g} = \frac{2u \cos \theta}{g}$$

$$\text{By multiplying } = t_1 t_2 = \frac{4u^2 \sin \theta \cos \theta}{g^2}$$

$$t_1 t_2 = \frac{2(u^2 \sin 2\theta)}{g^2} = \frac{2R}{g} \Rightarrow t_1 t_2 \propto R$$

9. (c) Total time of flight = $\frac{2u \sin \theta}{g} = \frac{2 \times 50 \times 1}{2 \times 10} = 5 \text{ s}$

Time to cross the wall = 3 sec (given)

Time in air after crossing the wall = $(5 - 3) = 2 \text{ sec}$

$\therefore$ Distance travelled beyond the wall = $(\cos \theta)t$

$$= 50 \times \frac{\sqrt{3}}{2} \times 2 = 86.6 \text{ m}$$

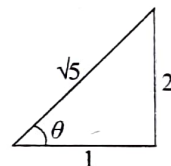
10. (a) $R = 2H$ given

$$\text{We know } R = 4H \cot \theta \Rightarrow \cot \theta = \frac{1}{2}$$

From triangle we can say that $\sin \theta = \frac{2}{\sqrt{5}}$, $\cos \theta = \frac{1}{\sqrt{5}}$

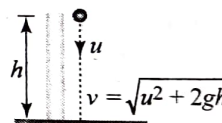
$$\therefore \text{Range of projectile } R = \frac{2v^2 \sin \theta \cos \theta}{g}$$

$$= \frac{2v^2}{g} \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{4v^2}{5g}$$

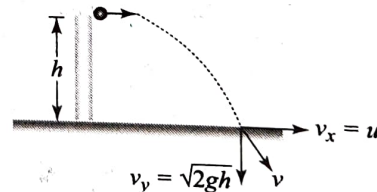


11. (c) When particle is thrown in vertical downward direction with velocity u , then final velocity at the ground level is

$$v^2 = u^2 + 2gh \therefore v = \sqrt{u^2 + 2gh}$$



Another particle is thrown horizontally with same velocity, then at the surface of earth.



Horizontal component of velocity $v_x = u$

$$\therefore \text{Resultant velocity, } v = \sqrt{u^2 + 2gh}$$

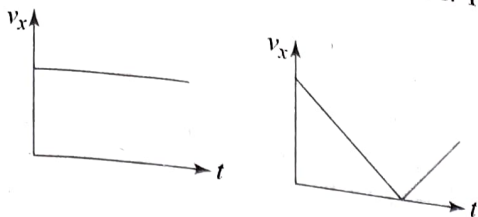
For both the particle final velocities when they reach the earth's surface are equal.

Multiple Correct Answers Type

12. (b, c, d)

At the highest point of the path, the particle has only horizontal component of velocity which is constant hence speed is

- minimum. Also at the highest point potential energy is maximum, so the kinetic energy will be minimum.
13. (b, c)
Magnitude of v_x remains constant. Magnitude of v_y decreases linearly with time to zero and then increases. Therefore the correct graphs are



Comprehension Type

Sol. 14. (b) 15. (b) 16. (c)

The time taken to reach maximum height and maximum height are

$$t = \frac{u \sin \theta}{g} \quad \text{and} \quad H = \frac{u^2 \sin^2 \theta}{2g}$$

For remaining half, the time of flight is

$$t' = \sqrt{\frac{2H}{g}} = \sqrt{\frac{u^2 \sin^2 \theta}{2g^2}} = \frac{t}{\sqrt{2}}$$

$$\therefore \text{Total time of flight is } t + t' = t \left(1 + \frac{1}{\sqrt{2}} \right)$$

$$T = \frac{u \sin \theta}{g} \left(1 + \frac{1}{\sqrt{2}} \right)$$

Also horizontal range is

$$= u \cos \theta \times T = \frac{u^2 \sin 2\theta}{2g} \left(1 + \frac{1}{\sqrt{2}} \right)$$

Let u_y and v_y be initial and final vertical components of velocity.

$$\therefore u_y^2 = 2gH \quad \text{and} \quad v_y^2 = 4gH$$

$$\therefore v_y = \sqrt{2} u_y$$

Angle (ϕ) final velocity makes with horizontal is

$$\tan \phi = \frac{v_y}{u_x} = \sqrt{2} \frac{u_y}{u_x} = \sqrt{2} \tan \theta$$

DPP 5.2

Single Correct Answer Type

1. (d)

$$\text{From graph } u_y = \frac{dy}{dt} = \tan 60^\circ$$

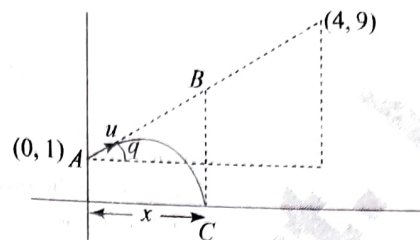
$$u_y = \tan 60^\circ = \sqrt{3} \text{ m/s}$$

$$\text{Range } R = \frac{2u_x u_y}{g} \quad \text{or} \quad \sqrt{3} = \frac{2 \times u_x \times \sqrt{3}}{g}$$

$$\text{or } u_x = 5 \text{ m/s}$$

$$\therefore u = \sqrt{u_x^2 + u_y^2} = \sqrt{28} \text{ m/s}$$

2. (c)



$$\tan \theta = \frac{9-1}{4-0} = 2$$

$$\text{Now, } -1 = u \sin \theta (1) - \frac{1}{2} g (1)^2$$

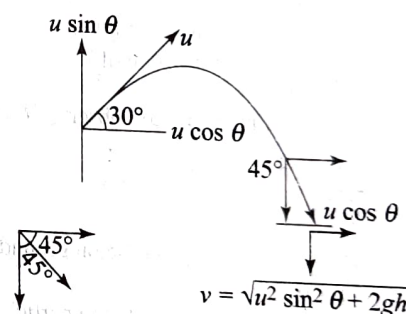
$$u \sin \theta = 4 \quad \text{and} \quad \sin \theta = \frac{2}{\sqrt{5}} \Rightarrow u = 2\sqrt{5}$$

$$\text{Now, } x = u \cos \theta (1) = (2\sqrt{5}) \times \frac{1}{\sqrt{5}} = 2 \text{ m}$$

3. (c) Using $v = \sqrt{u^2 + 2gh}$

$$v = \sqrt{u^2 \sin^2 \theta + 2gh}$$

(vertical comp. when striking)



$$\text{Now } \tan 45^\circ = 1$$

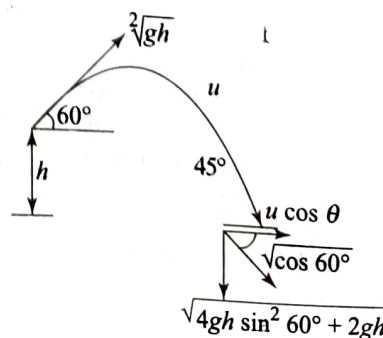
$$u \cos \theta = \sqrt{u^2 \sin^2 \theta + 2gh}$$

$$u^2 \cos^2 \theta = u^2 \sin^2 \theta + 2gh$$

$$u^2 \left(\frac{3}{4} - \frac{1}{4} \right) = 2gh$$

$$u^2 = 4gh \Rightarrow u = 2\sqrt{gh}$$

$$\tan \theta = \frac{V_{\perp}}{V_H} = \frac{\sqrt{4gh \cdot \frac{3}{4} + 2gh}}{2\sqrt{gh} \times \frac{1}{2}} = \frac{\sqrt{5gh}}{\sqrt{gh}} = \sqrt{5}$$

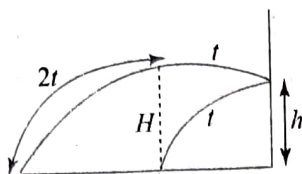


$$4. (c) \quad H = \frac{1}{2} g(2t)^2 = 2gt^2 \quad (1)$$

$$h = H - \frac{1}{2} gt^2 \quad (2)$$

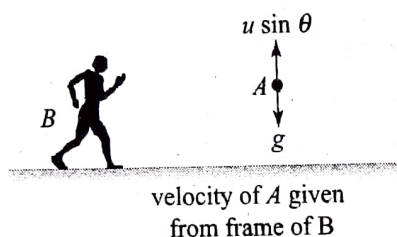
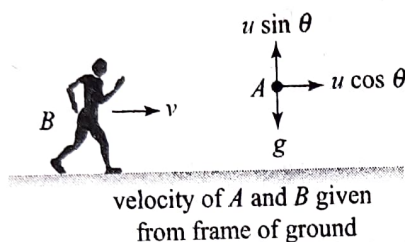
By (1) & (2)

$$h = H - \frac{H}{4} = \frac{3H}{4}$$



5. (a) The horizontal and vertical components of initial velocity of projectile are as shown in figure. Since the observer moving with uniform velocity v sees the projectile moving in straight line

Hence $v = u \cos \theta$



The time of flight as measured by observer B is

$$T = \frac{2u \sin \theta}{g}$$

Hence horizontal range of projectile on ground

$$R = (u \cos \theta)T = vT$$

6. (d) As the relative acceleration is zero, the particles will meet for all values of u_1 and u_2 (Assuming they do not collide with the ground. It is given that the cliffs are very high)

$$7. (c) \quad R = \frac{u^2 \sin 2\theta}{g} \text{ is same for angles } \theta \text{ \& } 90^\circ - \theta.$$

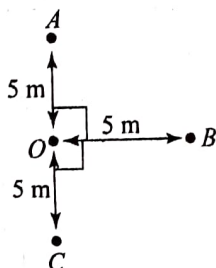
$$t_1 = \frac{2u \sin \theta_1}{g} = \frac{2u \sin(90^\circ - \theta_2)}{g}$$

$$t_2 = \frac{2u \sin \theta_2}{g} = \frac{2u \sin(90^\circ - \theta_1)}{g}$$

8. (c) Let the stones be projected at $t = 0$ sec with a speed u from point O. Then an observer, at rest at $t = 0$ and having constant acceleration equal to acceleration due to gravity, shall observe the three stones move with constant velocity as shown.

In the given time each ball shall travel a distance 5 metre as seen by this observer. Hence the required distance between A and B will be

$$= \sqrt{5^2 + 5^2} = 5\sqrt{2} \text{ metre}$$



9. (c)

Let initial and final speeds of stone be u and v .

$$\therefore v^2 = u^2 - 2gh$$

$$\text{and } v \cos 30^\circ = u \cos 60^\circ$$

solving 1 and 2 we get

$$u = \sqrt{3gh}$$

$$10. (c) \quad y = x \tan \theta \left(1 - \frac{x}{R}\right)$$

$$\Rightarrow \frac{R}{4} = \frac{3R}{4} \tan \theta \left(1 - \frac{3R}{4R}\right)$$

$$\Rightarrow 1 = 3 \tan \theta \left(\frac{1}{4}\right)$$

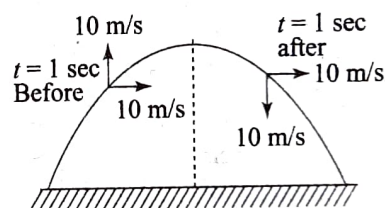
$$\Rightarrow \tan \theta = \frac{3}{4} \Rightarrow \theta = 53^\circ$$

11. (c) Displacement of aircraft in time t = horizontal displacement of projectile

$$\Rightarrow 972 \times \frac{5}{18} t = V_0 \cos \theta \cdot t$$

$$\Rightarrow \cos \theta = \frac{54 \times 5}{540} = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

12. (d) It is obvious from the figure shown that the angle between the velocity vectors 1 sec before and 1 sec after it attains maximum height is 90° .



13. (c) Acceleration of hero in vertical direction = 2 m/s^2
Acceleration of bullet in vertical direction = 10 m/s^2
Hence by the time bullet reaches the hero, its vertical displacement will be more than that of the hero.

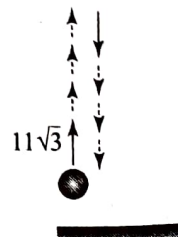
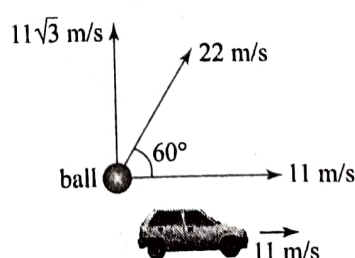
Multiple Correct Answers Type

14. (a, c)

w.r.t. to observer (V_x) football/obs.

$$= 11 - 11 = 0$$

So motion of the ball as seen from observer will be purely vertical with initial velocity $11\sqrt{3}$ upwards.



15. (b, c)

$$\text{Min. speed} = 10 \cos 30^\circ = 5\sqrt{3}$$

$$\frac{R}{2} = \frac{u^2 \sin 60^\circ}{2g} = \frac{5\sqrt{3}}{2}$$

$$H = \frac{u^2 \sin^2 30^\circ}{2g} = \frac{5}{4}$$

$$\text{Displacement} = \sqrt{H^2 + R^2} = \sqrt{\frac{75}{4} + \frac{25}{16}} = \frac{1}{4}\sqrt{325}$$

$$v_y \text{ (2 sec)} \\ = 10 \sin 30^\circ - 10 \times 2 \\ v_y = -15 \text{ m/s}$$

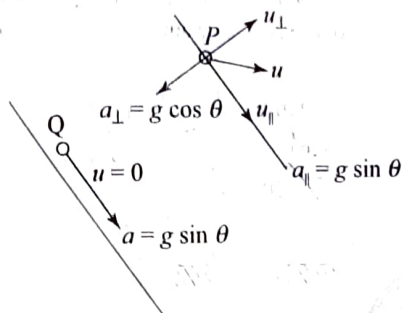
$$T = \frac{2u \sin \theta}{g} = 1 \text{ sec}$$

Thus, before 2 sec particle will strike the floor.

DPP 5.3

Single Correct Answer Type

1. (b) It can be observed from figure that P and Q shall collide if the initial component of velocity of P along incline, $u_{\parallel} = 0$ that is particle is projected perpendicular to incline.



$\therefore$ Time of flight

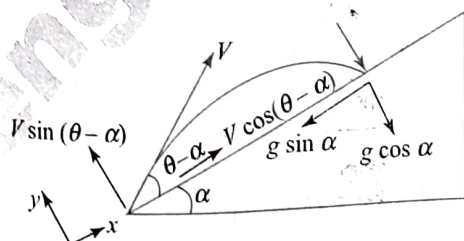
$$T = \frac{2u_{\perp}}{g \cos \theta} = \frac{2u}{g \cos \theta}$$

$$\therefore u = \frac{gT \cos \theta}{2} = 10 \text{ m/s}$$

2. (d) Applying equation of motion perpendicular to the incline for $y = 0$,

$$0 = V \sin(\theta - \alpha)t + \frac{1}{2}(-g \cos \alpha)t^2$$

$$t = 0 \quad \& \quad \frac{2V \sin(\theta - \alpha)}{g \cos \alpha}$$



At the moment of striking the plane, as velocity is perpendicular to the inclined plane hence component of velocity along incline must be zero.

$$0 = v \cos(\theta - \alpha) + (-g \sin \alpha)t$$

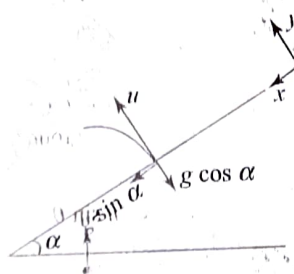
$$\frac{2V \sin(\theta - \alpha)}{g \cos \alpha}$$

$$v \cos(\theta - \alpha) = \tan \alpha \cdot 2v \sin(\theta - \alpha) \\ \cot(\theta - \alpha) = 2 \tan \alpha$$

3. (a) Equation for motion perpendicular to the incline

$$\Rightarrow u \cdot t - \frac{1}{2}g \cos \alpha \cdot t^2 \Rightarrow t = \frac{2u}{g \cos \alpha}$$

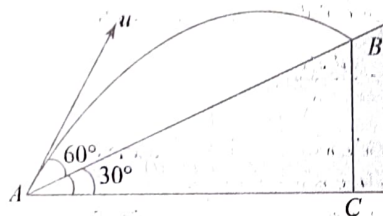
$$\text{Along incline } x = 0 + \frac{1}{2} \times (g \sin \alpha) \left(\frac{2u}{g \cos \alpha} \right)^2$$



4. (a) The horizontal displacement in time t is

$$AC = u \cos 60^\circ t = \frac{ut}{2}$$

$$\therefore \text{Range on inclined plane} = \frac{AC}{\cos 30^\circ} = \frac{ut}{\sqrt{3}}$$



5. (c) After the elastic collision with inclined plane the projectile moves in vertical direction.

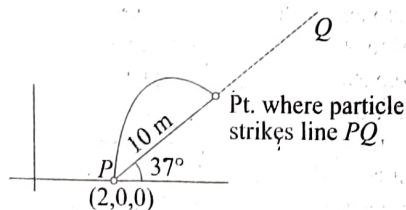
The inclination of plane with horizontal is 45° , hence velocity of particle just before collision should be horizontal.

$\therefore$ Time required to reach maximum height

$$= t_{AB} + t_{BC} = \frac{u \sin \theta}{g} + \frac{u \cos \theta}{g}$$

6. (a) Range = 10 m.

For point where particle strikes line PQ



$$\therefore \begin{aligned} x \text{ coordinate} &= 10 \cos 37^\circ + 2 = 10 \text{ m} \\ y \text{ coordinate} &= 10 \sin 37^\circ = 6 \text{ m} \\ z \text{ coordinate} &= 0 \text{ m} \end{aligned}$$

7. (a) Time of flight $t = \frac{2u \sin(2\theta - \theta)}{g \cos \theta} = \frac{2u}{g} \tan \theta$

$$AB = \frac{2u^2 \tan \theta}{g\sqrt{3}} = \frac{u}{\sqrt{3}} t$$

$$u_H = u \cos 2\theta$$

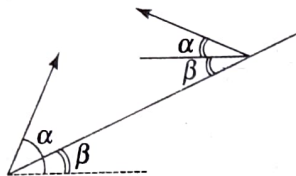
$$AC = u_H \cdot t = u \cos 2\theta t$$

$$AB = \frac{AC}{\cos \theta} = \frac{u \cos 2\theta t}{\cos \theta}$$

Compare equations (i) and (ii)

$$\frac{\cos 2\theta}{\cos \theta} = \frac{1}{\sqrt{3}} \text{ or } \theta = 30^\circ$$

8. (d) $t_A = \frac{2u \sin(\alpha + \beta)}{g \cos \beta}$ and $t_B = \frac{2u \sin(\alpha - \beta)}{g \cos \beta}$



$$\frac{t_A}{t_B} = \frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)} = \frac{\cos \theta}{\sin \theta} = \frac{\sin(90 - \theta)}{\sin \theta}$$

$$\alpha + \beta = 90 - \theta \Rightarrow \alpha = 45^\circ$$

Multiple Correct Answers Type

9. (a, c, d)

$$R_1 = \frac{24^2 \sin \alpha \cos(\alpha + \theta)}{g \cos^2 \theta}$$

$$R_2 = \frac{24^2 \sin \alpha \cos(\alpha - \theta)}{g \cos^2 \theta}$$

$$R_1 - R_2 = g \sin \theta T_2^2$$

$$R_1 - R_2 = g \sin \theta T_1^2$$

Matching Column Type

10. (a) $R = \frac{u^2 \sin 2\theta}{g} = \frac{100\sqrt{3}}{2(10)} = 5\sqrt{3} \text{ m}$

(b) $11.25 = -10 \sin 60^\circ t + \frac{1}{2}(10)t^2$

$$\Rightarrow 5t^2 - 5\sqrt{3}t - 11.25 = 0$$

$$t = \frac{5\sqrt{3} \pm \sqrt{25(3) + 4(5)(11.25)}}{10}$$

$$= \frac{5\sqrt{3} \pm \sqrt{3}(10)}{10} = \frac{15}{10}\sqrt{3} = \frac{3}{2}\sqrt{3}$$

(c) $R = 10 \cos 60^\circ \left(\frac{3}{2}\sqrt{3} \right) = 7.5\sqrt{3} \text{ m}$

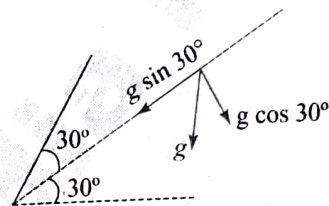
$$t = \frac{2u \sin 30^\circ}{g \cos 30^\circ} = \frac{2(10)\left(\frac{1}{2}\right)}{10\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}} \text{ sec}$$

(ii)

$$R = 10 \cos 30^\circ t - \frac{1}{2} g \sin 30^\circ t^2$$

$$= \frac{10\sqrt{3}}{2} \left(\frac{2}{\sqrt{3}} \right) - \frac{1}{2}(10)\left(\frac{1}{2}\right)\frac{4}{3}$$

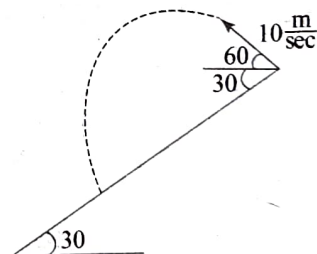
$$= 10 - \frac{10}{3} = \frac{20}{3} \text{ m}$$



(d) $T = \frac{2(10)}{g \cos 30^\circ} = \frac{2(10)}{10\left(\frac{\sqrt{3}}{2}\right)} = \frac{4}{\sqrt{3}} \text{ sec.}$

$$R = \frac{1}{2} g \sin 30^\circ t^2$$

$$= \frac{1}{2}(10)\left(\frac{1}{2}\right)\frac{16}{3} = \frac{40}{3} \text{ m}$$



Comprehension Type

Sol. 11 (a), 12 (c), 13 (b)

11. (a) Let us be the velocity of ball A is 2.0 m.

$$2.0 = (u \sin \theta)t - \frac{1}{2}gt^2$$

For bullet C, vertical distance travelled $= u \sin 30^\circ t + \frac{1}{2}gt^2$ (i)

For bullet C, vertical distance travelled

$$d = u't - \frac{1}{2}gt^2 \quad \text{(ii)}$$

From equation (i) and (ii),

$$u \sin 30^\circ t + u't = 1500$$

$$50t + 100t = 1500 \text{ which gives } t = 10 \text{ sec}$$

12. (c) CO = greatest height for bullet A

$$CO = u't - \frac{1}{2}gt^2 = 100(10) - \frac{1}{2}(10)^3 = 500$$

$$AC = \frac{1}{2}(\text{Range for bullet at A})$$

$$AC = u \cos 30^\circ t = 500\sqrt{3}$$

$$\tan \phi = \frac{CO}{AC} = \frac{1}{\sqrt{3}} \Rightarrow \phi = 30^\circ$$

13. (b) If θ is the angle of projection at A

$$\tan \theta = 2 \tan \phi = 2\sqrt{3}$$

$$\cos \theta = \frac{1}{\sqrt{7}}$$

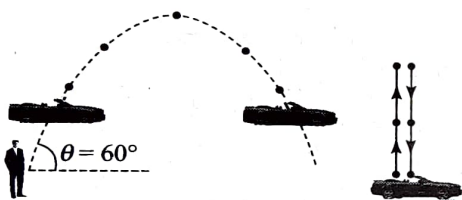
For bullet at A, $u_A \cos \theta = 500\sqrt{3}$

$$u_A = \frac{500\sqrt{3}(\sqrt{7})}{\sqrt{3} \times 10} = 50\sqrt{7} \text{ m/s}$$

DPP 5.4

Subjective Type

1. Consider the diagram below



The boy throws the ball at an angle of 60° .

$\therefore$ Horizontal component of velocity = $4 \cos \theta$

$$= (10 \text{ m/s}) \cos 60^\circ = 10 \times \frac{1}{2} = 5 \text{ m/s}$$

Speed of the car = $18 \text{ km/h} = 5 \text{ m/s}$

As horizontal speed of ball and car is same, hence relative velocity of car and ball in the horizontal direction will be zero. Only vertical motion of the ball will be seen by the boy in the car, as shown in the figure

2. Let us draw the vector diagram of the information given and find a and b . We may draw all vectors in the reference frame of ground-based observer. Assume north to be $\hat{i}$ direction and vertically downward to be $-\hat{j}$.

Let the rain velocity v_r be $a\hat{i} + b\hat{j}$.

$$v_r = a\hat{i} + b\hat{j}$$

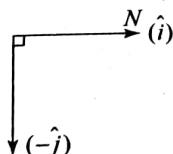
Case I Given velocity of girl =

$$v_g = (5 \text{ m/s}) \hat{i}$$

Let v_{rg} = velocity of rain w.r.t girl

$$= v_r - v_g = (a\hat{i} + b\hat{j}) - 5\hat{i}$$

$$= (a-5)\hat{i} + b\hat{j}$$



According to question rain, appears to fall vertically downward.

Hence, $a-5=0 \Rightarrow a=5$

Case II Given velocity of the girl,

$$v_g = (10 \text{ m/s}) \hat{i}$$

$$\therefore v_{rg} = v_r - v_g$$

$$= (a\hat{i} + b\hat{j}) - 10\hat{i} = (a-10)\hat{i} + b\hat{j}$$

According to question rain appears to fall at 45° to the vertical

$$\text{Hence } \tan 45^\circ = \frac{b}{a-10} = 1$$

$$\Rightarrow b = a - 10 = 5 - 10 = -5$$

Hence, velocity of rain = $a\hat{i} + b\hat{j}$

$$\Rightarrow v_r = 5\hat{i} - 5\hat{j}$$

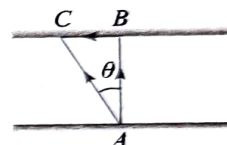
Speed of rain

$$= |v_r| = \sqrt{(5)^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2} \text{ m/s}$$

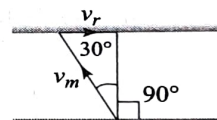
Single Correct Answer Type

3. (a)

For shortest time, swimmer should swim along AB, so he will reach at point C due to the velocity of river. i.e. he should swim due north.



4. (c)



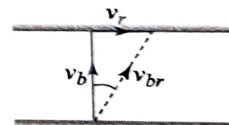
$$\sin 30^\circ = \frac{v_r}{v_m} = \frac{1}{2} \Rightarrow v_r = \frac{v_m}{2} = \frac{0.5}{2} = 0.25 \text{ m/s}$$

5. (b) $\vec{v}_{br} = \vec{v}_b + \vec{v}_r$

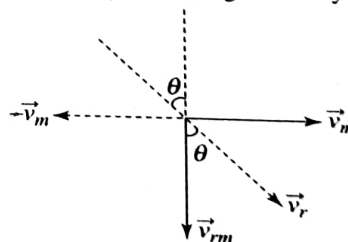
$$\Rightarrow v_{br} = \sqrt{v_b^2 + v_r^2}$$

$$\Rightarrow 10 = \sqrt{8^2 + v_r^2}$$

$$\Rightarrow v_r = 6 \text{ km/hr}$$



6. (b) A man is sitting in a bus and travelling from west to east, and the raindrops are appears falling vertically down.



v_m = velocity of man

v_r = Actual velocity of rain which is falling at an angle θ with vertical

v_{rm} = velocity of rain w.r.t. to moving man

If the another man observes the rain then he will find that actually rain is falling with velocity v_r at an angle going from west to east.

7. (b) $V_{\text{boat, river}} = 4.5 \text{ km/m}$
 $V_{\text{river, ground}} = 6 \text{ km/hr.}$

$$V_{\text{boat, ground}} = (6\hat{i} + 4.5\hat{j}) \text{ km/m} = (6\hat{i} + 4.5\hat{j}) \times \frac{5}{18} \text{ m}$$

$$V_{\text{boat, ground}} = \sqrt{6^2 + 4.5^2} \times \frac{5}{18}$$

$$= 1.5 \times 5 \times \frac{5}{18} = \frac{37.5}{18} = 2.1 \text{ m/sec}$$

8. (b) Time taken by man to cross the river = $\frac{\text{width of river}}{v_y}$

$$12 = \frac{60}{v_y} \Rightarrow v_y = 5 \text{ m/sec}$$

Let the x component of velocity of man w.r. to river is v_x

Since velocity of man w.r. to ground makes an angle of 45° with river flow x component of velocity of man w.r. to ground

= y component of velocity of man w.r. to ground

$$v_r + v_x = v_y$$

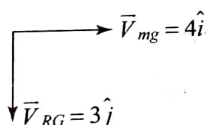
$$v_x = 0$$

So velocity of man w.r. to water = $v_y = 5 \text{ m/sec}$

9. (b) $\vec{V}_A = -500\hat{i} \Rightarrow \vec{V}_{GA} + \vec{V}_A = V_{GA}$
 $\vec{V}_{GA} = 1500\hat{i} \Rightarrow 1500\hat{i} - 500\hat{i} = 1000\hat{i}$



10. (a) $\vec{V}_{RG} = 3\hat{j} - 4\hat{i}$
 $V_{RG} = 5 \text{ km/sec.}$



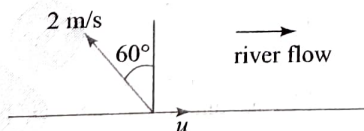
11. (a) Let speed of current be u .

For net velocity of man to be

normal to river flow.

$$2 \cos 60^\circ = u$$

or $u = 1 \text{ km/hr.}$



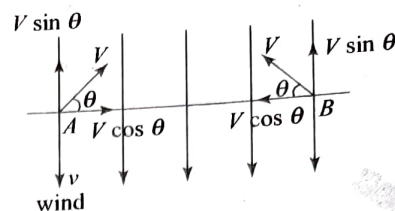
DPP 5.5

Single Correct Answer Type

1. (a) For no drift: $V \sin \theta = v$

$$\sin \theta = \frac{v}{V} \therefore t = t_{AB} + t_{BA}$$

$$t = \frac{2\ell}{V \cos \theta} = \frac{2\ell}{V \sqrt{1 - \frac{v^2}{V^2}}} \Rightarrow t = \frac{2\ell}{\sqrt{V^2 - v^2}}$$

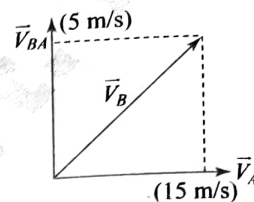


2. (b) Velocity of A and velocity of B relative to A are as shown.

$$\therefore \vec{V}_B = \vec{V}_{BA} + \vec{V}_A$$

$$\Rightarrow V_0 = \sqrt{5^2 + 15^2} = 5\sqrt{10} \text{ m/s}$$

and direction of $\vec{V}_B$ is away from $\vec{V}_A$.



3. (c) Relative to the person in the train, acceleration of the stone is 'g' downward, a (acceleration of train) backwards.

$$\text{According to him: } x = \frac{1}{2}at^2, \quad Y = \frac{1}{2}gt^2$$

$$\Rightarrow \frac{X}{Y} = \frac{a}{g} \Rightarrow Y = \frac{g}{a}X \text{ straight line}$$

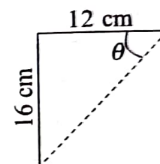
4. (a) $V_{R/G(x)} = 0, V_{R/G(y)} = 10 \text{ m/s}$
 Let, velocity of man = v

$$\tan \theta = \frac{16}{12} = \frac{4}{3}$$

Then $v_{R/\text{man}} = v$ (opposite to man)
 For the required condition:

$$\tan \theta = \frac{V_{R/M(y)}}{V_{R/M(x)}} = \frac{10}{v} = \frac{4}{3}$$

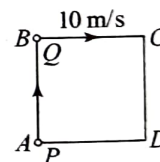
$$\Rightarrow V = \frac{10 \times 3}{4} = 7.5 \text{ m/s}$$



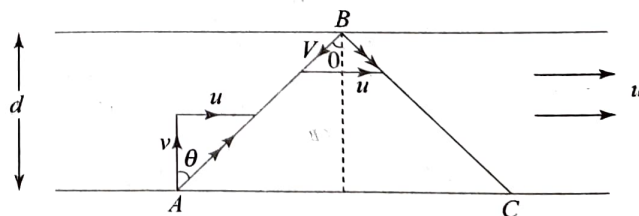
5. (b) $a = 8 \text{ m}$

They meet when Q displace $8 \times 3 \text{ m}$ more that $\Rightarrow$ relative displacement = relative velocity $\times$ time.

$$8 \times 3 = (10 - 2)t \Rightarrow t = 3 \text{ sec}$$



6. (c)



V = velocity of man w.r.t. river

u = velocity of river

$$A \rightarrow B = \frac{d}{v} \Rightarrow 10 = \frac{d}{v} \Rightarrow d = 10V \quad (i)$$

$$B \rightarrow C = \frac{d}{v \cos \theta} \Rightarrow 15 = \frac{d}{v \cos \theta}$$

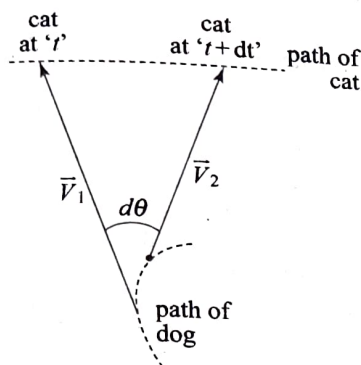
$$\Rightarrow d = 15 v \cos \theta \rightarrow \quad (ii)$$

$$(i) \& (i) \Rightarrow \cos \theta = 2/3 \Rightarrow \sec \theta = 3/2$$

$$\Rightarrow \tan \theta = \frac{u}{v} \therefore \sqrt{\sec^2 \theta - 1} = \frac{u}{v}$$

$$\frac{u}{v} = \sqrt{9/4 - 1} = \frac{\sqrt{5}}{2} \Rightarrow \frac{v}{u} = \frac{2}{\sqrt{5}}$$

7. (a) The diagram shows the dog's velocity just before and just after the moment in question, with a very small angle between them. Extending those arrows to the path of cat forms an isosceles triangle with height x and width $v dt$. ($\therefore$ velocity of dog is always directed towards cat.)



Magnitude of acceleration of dog at required moment

$$= \frac{|\vec{v}_2 - \vec{v}_1|}{dt}$$

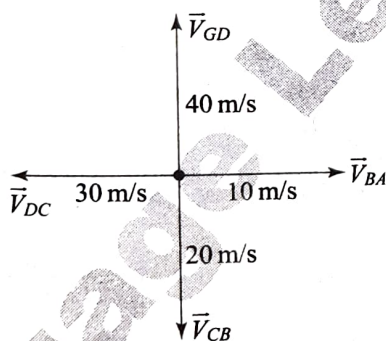
$$\text{Now } |\vec{v}_2 - \vec{v}_1| = u d\theta$$

$$\text{and } d\theta = \frac{v dt}{x}$$

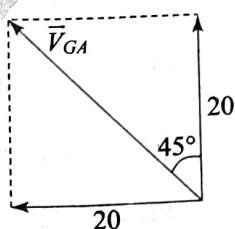
From equation (i), (ii) and (iii),

$$\text{required magnitude of acceleration} = \frac{uv}{x}$$

8. (c) All the velocities are marked in diagram where G represents ground



Adding we get



$$\text{Then } \vec{V}_{GD} + \vec{V}_{DC} + \vec{V}_{CB} + \vec{V}_{BA} = \vec{V}_{GA} = -\vec{V}_{AG}$$

Hence velocity of A is towards south east.

9. (b) If the water is at rest both the particles will be diametrically opposite on the circle after 8 seconds. The distance between them remains same even if the water is flowing. Hence the distance between them after 8 seconds is $2 \times 10 = 20$ m.

$$10. (c) \vec{v}_{RM} = \vec{v}_R - \vec{v}_m$$

$$\vec{v}_{RM} = \vec{v}_R - u\hat{i} = (V_{RX}\hat{i} + V_{RY}\hat{j}) - u\hat{i}$$

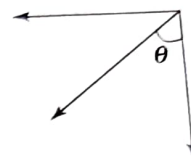
Since $\vec{v}_{RM}$ has only y component with respect to the man.

$$\text{so, } V_{RX} - u = 0$$

$$\Rightarrow V_{RX} = u$$

After doubling the speed

$$\begin{aligned} \vec{v}_{Rm'} &= (u\hat{i} + v_{Ry}\hat{j}) - 2u\hat{i} \\ &= -u\hat{i} + v_{Ry}\hat{j} \end{aligned}$$



$$\text{Given } \tan \theta = \frac{u}{v_{Ry}} \Rightarrow v_{Ry} = u \cot \theta$$

$$\text{so } \vec{v}_R = u\hat{i} + u \cot \theta (\hat{j})$$

11. (d) Let $\hat{i}$ and $\hat{j}$ be unit vectors in direction of east and north respectively.

$$\therefore \vec{V}_{DC} = 20\hat{j}, \vec{V}_{BC} = 20\hat{i} \text{ and } \vec{V}_{BA} = -20\hat{j}$$

(i)

$$\therefore \vec{V}_{DA} = \vec{V}_{DC} + \vec{V}_{CB} + \vec{V}_{BA} = 20\hat{j} - 20\hat{i} - 20\hat{j} = -20\hat{i}$$

(ii)

$$\therefore \vec{V}_{DA} = -20\hat{i}$$

(iii)

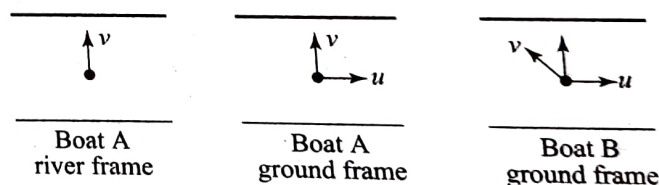
Multiple Correct Answers Type

12. (b, d)

Speed of river is u and speed of boat relative to water is v .

$$\text{Speed of boat A observed from ground} = \sqrt{u^2 + v^2}$$

$$\text{Speed of boat B observed from ground} = \sqrt{v^2 - u^2}$$



From river frame, speed of boat A and B will be same.

Comprehension Type

13. (b) To reach the port B, the x-component of the total velocity must be zero: $v \sin \theta - u = 0$.

$$\text{So } \sin \theta = 1/2 \text{ or } \theta = 30^\circ$$

Take positive x-axis along east and positive y-axis along north.

14. (b) To cross the river the fastest, we need to maximize the y-component of the total velocity is $v_B \cos \theta$. So $\theta = 0$. The boat should head straight to the north.

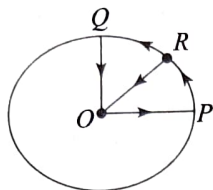
Take positive x-axis along east and positive y-axis along north.

15. (d) The trip takes time $t = \frac{D}{v}$. The y-component of the total velocity is u . So the boat is at a distance $u \frac{D}{v} = \frac{D}{2}$ downstream from the port B after crossing.
Take positive x-axis along east and positive y-axis along north.

DPP 5.6

Subjective Type

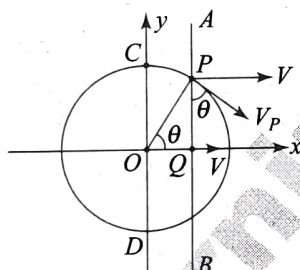
1. As shown in the adjacent figure. The cyclist covers the path $OPRQO$.
As we know whenever an object performing circular motion, acceleration is called centripetal acceleration and is always directed towards the centre.



Hence, acceleration at R is $a = \frac{v^2}{r}$

$$\Rightarrow a = \frac{(10)^2}{1 \text{ km}} = \frac{100}{10^3} = 0.1 \text{ m/s}^2 \text{ along } RO.$$

2. (a) As a rod AB moves, the point 'P' will always lie on the circle.
Hence its velocity will be along the circle as shown by ' V_P ' in the figure.
If the point P has to lie on the rod 'AB' also then it should have component in 'x' direction as ' V '.



$$V_P \sin \theta = V \Rightarrow V_P = V \operatorname{cosec} \theta$$

$$\text{Here } \cos \theta = \frac{x}{R} = \frac{1}{R} \cdot \frac{3R}{5} = \frac{3}{5}$$

$$\therefore \sin \theta = \frac{4}{5} \Rightarrow \operatorname{cosec} \theta = \frac{5}{4}$$

$$\Rightarrow V_P = \frac{5}{4}V$$

- (b) And angular velocity $\omega = \frac{V_P}{R} = \frac{5V}{4R}$

Alternative Solution:

- (a) Let 'P' have coordinate (x, y)
 $x = R \cos \theta$, $y = r \sin \theta$.

$$V_x = \frac{dx}{dt} = -R \sin \theta \frac{d\theta}{dt} = V$$

$$\frac{d\theta}{dt} = \frac{-V}{R \sin \theta} \text{ and } V_y = R \cos \theta \frac{d\theta}{dt}$$

$$= R \cos \theta \left(-\frac{V}{R \sin \theta} \right) = -V \cot \theta$$

$$V_P = \sqrt{V_x^2 + V_y^2} \\ = \sqrt{V^2 + V^2 \cot^2 \theta} = V \operatorname{cosec} \theta$$

(b) $\omega = \frac{V_P}{R} = \frac{5V}{4R}$

Single Correct Answer Type

3. (c) Both changes in direction although their magnitudes remains constant.

4. (c) Angular acceleration (α) = $\frac{a_t}{r}$

$$\text{Since, } |\vec{a}_t| = \frac{d|\vec{v}|}{dt} = \text{constant}$$

$\therefore$ Magnitude of α is constant

Also its direction is always constant (perpendicular to the plane of circular motion). Whereas direction changes continuously $\vec{a}_t$ is not constant

5. (c) Change in velocity = $2v \sin(\theta/2) = 2v \sin 20^\circ$

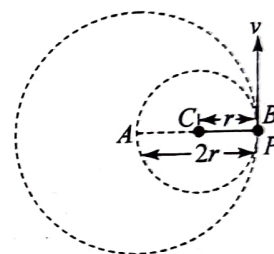
6. (b) Angular velocity of particle P about point A,

$$\omega_A = \frac{v}{r_{AB}} = \frac{v}{2r}$$

Angular velocity of particle P about point C,

$$\omega_C = \frac{v}{r_{BC}} = \frac{v}{r}$$

$$\text{Ratio } \frac{\omega_A}{\omega_C} = \frac{v/2r}{v/r} = \frac{1}{2}$$



7. (c) $\omega_{\min} = \frac{2\pi \text{ Rad}}{60 \text{ min}}$ and $\omega_{hr} = \frac{2\pi \text{ Rad}}{12 \times 60 \text{ min}}$

$$\therefore \frac{\omega_{\min}}{\omega_{hr}} = \frac{2\pi/60}{2\pi/12 \times 60}$$

8. (b) $\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix} = -18\hat{i} - 13\hat{j} + 2\hat{k}$

9. (b) By using equation $\omega^2 = \omega_0^2 - 2\alpha\theta$

$$\left(\frac{\omega_0}{2} \right)^2 = \omega_0^2 - 2\alpha(2\pi n) \Rightarrow \alpha = \frac{3}{4} \frac{\omega_0^2}{4\pi \times 36}, (n=36) \quad (i)$$

Now let fan completes total n' revolution from the starting to come to rest

$$0 = \omega_0^2 - 2\alpha(2\pi n') \Rightarrow n' = \frac{\omega_0^2}{4\alpha\pi}$$

Substituting the value of α from equation (i)

$$n' = \frac{\omega_0^2}{4\pi} \frac{4 \times 4\pi \times 36}{3\omega_0^2} = 48 \text{ revolutions}$$

$$\text{Number of rotation} = 48 - 36 = 12$$

10. (b) Net acceleration in non-uniform circular motion,

$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(2)^2 + \left(\frac{900}{500}\right)^2} = 2.7 \text{ m/s}^2$$

a_t = tangential acceleration

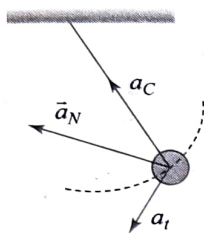
a_c = centripetal acceleration = $\frac{v^2}{r}$

11. (b) $a_{\text{resultant}} = \sqrt{a_{\text{radial}}^2 + a_{\text{tangential}}^2} = \sqrt{\frac{v^4}{r^2} + a^2}$

12. (c) a_c = centripetal acceleration

a_t = tangential acceleration

a_N = net acceleration = Resultant of a_c and a_t



13. (c) Tangential acceleration = $a_t = g \sin \theta$

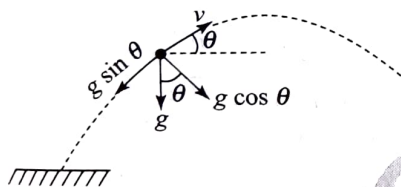
Normal acceleration = $a_n = g \cos \theta$

$$\Rightarrow a_t = a_n$$

$$g \sin \theta = g \cos \theta \Rightarrow \theta = 45^\circ$$

$$\Rightarrow v_y = v_x \Rightarrow u_y - g t = u_x$$

$$20 - (10)t = 10 \Rightarrow t = 1 \text{ sec}$$



During downward motion

$$a_t = a_n \Rightarrow v_y = -v_x$$

$$20 - 10t = -10 \Rightarrow t = 3 \text{ sec}$$

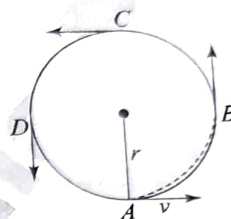
Multiple Correct Answers Type

14. (c, d)

In the given condition, the particle undergoes **uniform** circular motion and for uniform circular motion the velocity and acceleration vector **changes continuously** but kinetic energy is constant for every point.

15. (a, b, c)

For a particle performing uniform circular motion



- (i) speed will be constant throughout.
- (ii) velocity will be tangential in the direction of motion at a particular point.
- (iii) acceleration $a = \frac{v^2}{r}$ will always be towards centre of the circular path.
- (iv) Speed is constant, hence tangential acceleration is zero.

CHAPTER 6

DPP 6.1

Subjective Type

1. Consider figure (a)

$$\begin{aligned} v_x &= 2t \text{ for } 0 < t < 1 \text{ s} \\ &= 2(2-t) \text{ for } 1 < t < 2 \text{ s} \\ &= 0 \text{ for } t > 2 \text{ s} \end{aligned}$$

From figure (b) $v_y = t$ for $0 < t < 1$ s
 $= 1$ for $t > 1$ s

$$\begin{aligned} \therefore F_x &= ma_x = m \frac{dv_x}{dt} \\ &= 1 \times 2 = 2 \text{ N for } 0 < t < 1 \text{ s} \\ &= 1(-2) = -2 \text{ N for } 1 < t < 2 \text{ s} \\ &= 0 \text{ for } t > 2 \text{ s} \end{aligned}$$

$$\begin{aligned} F_y &= ma_y = m \frac{dv_y}{dt} \\ &= 1 \times 1 = 1 \text{ N for } 0 < t < 1 \text{ s} \\ &= 0 \text{ for } t > 1 \text{ s} \end{aligned}$$

Hence, $F = F_x \hat{i} + F_y \hat{j}$
 $= (2\hat{i} + \hat{j}) \text{ N for } 0 < t < 1 \text{ s}$
 $= -2\hat{i} \text{ N for } 1 < t < 2 \text{ s}$
 $= 0 \text{ for } t > 2 \text{ s}$

Single Correct Answer Type

2. (b) Horizontal velocity of apple will remain same but due to retardation of train, velocity of train and hence velocity of boy w.r.t. ground decreases, so apple falls away from the hand of boy in the direction of motion of the train.
3. (d) Horizontal velocity of ball and person are same so both will cover equal horizontal distance in a given interval of time and after following the parabolic path the ball falls exactly in the hand which threw it up.
4. (a) When the spring C is stretched slowly, the tension in A is greater than that of C, because of the weight mg and the former reaches breaking point earlier.
5. (b) u = velocity of bullet

$$\begin{aligned} \frac{dm}{dt} &= \text{Mass thrown per second by the machine gun} \\ &= \text{Mass of bullet} \times \text{Number of bullet fired per second} \\ &= 10 \text{ g} \times 10 \text{ bullet/sec} = 100 \text{ g/sec} = 0.1 \text{ kg/sec} \end{aligned}$$

$$\therefore \text{Thrust} = \frac{u dm}{dt} = 500 \times 0.1 = 50 \text{ N}$$

6. (d) u = velocity of bullet

$$\frac{dm}{dt} = \text{Mass fired per second by the gun}$$

$$\frac{dm}{dt} = \text{Mass of bullet } (m_B) \times \text{Bullets fired per sec } (N)$$

$$\text{Maximum force that man can exert } F = u \left(\frac{dm}{dt} \right)$$

$$\therefore F = u \times m_B \times N$$

$$\Rightarrow N = \frac{F}{m_B \times u} = \frac{144}{40 \times 10^{-3} \times 1200} = 3$$

7. (b) Velocity between $t = 0$ and $t = 2$ sec

$$\Rightarrow v_i = \frac{dx}{dt} = \frac{4}{2} = 2 \text{ m/s}$$

$$\text{Velocity at } t = 2 \text{ sec, } v_f = 0$$

$$\begin{aligned} \text{Impulse} &= \text{Change in momentum} = m(v_f - v_i) \\ &= 0.1(0 - 2) = -0.2 \text{ kg m sec}^{-1} \end{aligned}$$

8. (b) Displacement of body in 4 sec along OE

$$s_x = v_x t = 3 \times 4 = 12 \text{ m}$$

$$\text{Force along OF (perpendicular to OE)} = 4 \text{ N}$$

$$\therefore a_y = \frac{F}{m} = \frac{4}{2} = 2 \text{ m/s}^2$$

$$\text{Displacement of body in 4 sec along OF}$$

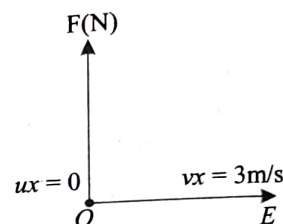
$$\Rightarrow s_y = u_y t + \frac{1}{2} a_y t^2 = \frac{1}{2} \times 2 \times (4)^2 = 16 \text{ m} \quad [\text{As } u_y = 0]$$

$$\therefore \text{Net displacement } s = \sqrt{s_x^2 + s_y^2} = \sqrt{(12)^2 + (16)^2} = 20 \text{ m}$$

9. (a) Given that $\vec{p} = p_x \hat{i} + p_y \hat{j} = 2 \cos t \hat{i} + 2 \sin t \hat{j}$

$$\therefore \vec{F} = \frac{d\vec{p}}{dt} = -2 \sin t \hat{i} + 2 \cos t \hat{j}$$

$$\text{Now, } \vec{F} \cdot \vec{p} = 0 \text{ i.e. angle between } \vec{F} \text{ and } \vec{p} \text{ is } 90^\circ.$$



Multiple Correct Answers Type

10. (a, b, d)

$$\text{Given, } x = 0 \text{ for } t < 0 \text{ s}$$

$$x(t) = A \sin 4\pi t; \text{ for } 0 < t < \frac{1}{4} \text{ s}$$

$$x = 0; \text{ for } t > \frac{1}{4} \text{ s}$$

$$\text{For, } 0 < t < \frac{1}{4} \text{ s } v(t) = \frac{dx}{dt} = 4\pi A \cos 4\pi t$$

$$a(t) = \text{acceleration}$$

$$= \frac{dv(t)}{dt} = -16\pi^2 A \sin 4\pi t$$

$$\text{At } t = \frac{1}{8} \text{ s, } a(t) = -16\pi^2 A \sin 4\pi \times \frac{1}{8} = -16\pi^2 A$$

$$F = ma(t) = -16\pi^2 A \times m = -16\pi^2 \text{ mA}$$

$$\text{Impulse} = \text{Change in linear momentum}$$

$$= F \times t = (-16\pi^2 \text{ Am}) \times \frac{1}{4} = -4\pi^2 \text{ Am}$$

The impulse (Change in linear momentum)

at $t = 0$ is same as, $t = \frac{1}{4} \text{ s}$

Clearly, force depends upon A which is not constant. Hence, force is also not constant.

11. (c, d)

Given, $m_1 = m_2 = 50(\text{g}) = \frac{1}{20} \text{ kg}$

Initial velocity (u) $u_1 = u_2 = 5 \text{ m/s}$

Final velocity (v) $v_1 = v_2 = -5 \text{ m/s}$

Time duration of collision $= 10^{-3} \text{ s}$

Change in linear momentum, $m(v - u) = \frac{1}{20} [-5 - 5] = -0.5 \text{ N-s}$

$$\text{Force} = \frac{\text{impulse}}{\text{Time}} = \frac{\text{change in momentum}}{10^{-3} \text{ s}}$$

$$= \frac{0.5}{10^{-3}} = 500 \text{ N}$$

Impulse and force are opposite in directions.

Matching Column Type

12. (p $\rightarrow$ a, b, c), (q $\rightarrow$ a, b, c), (r $\rightarrow$ a), (s $\rightarrow$ d)

(p) $\vec{F} = \frac{d\vec{p}}{dt} = \text{constant} \Rightarrow \vec{p}$ must change its magnitude and may/may not change its direction.

(q) This is similar to previous case.

(r) Since $\vec{F}$ is changing in direction only, $\vec{p}$ may change its direction.

(s) $\vec{F} = \frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p} = \text{constant}$

DPP 6.2

Subjective Type

1. Given, mass of helicopter (m_1) = 2000 kg
Mass of the crew and passengers $m_2 = 500 \text{ kg}$
Acceleration in vertical direction $a = 15 \text{ m/s}^2$ ($\uparrow$) and $g = 10 \text{ m/s}^2$ ($\downarrow$)

(a) Force on the floor of the helicopter by the crew and passengers

$$m_2(g + a) = 500(10 + 15) \text{ N}$$

$$500 \times 25 \text{ N} = 12500 \text{ N}$$

(b) Action of the rotor of the helicopter on the surrounding

$$\text{air} = (m_1 + m_2)(g + a)$$

$$= (2000 + 500) \times (10 + 15) = 2500 \times 25$$

$$= 62500 \text{ N (downward)}$$

(c) Force on the helicopter due to the surrounding air

= reaction of force applied by helicopter.

$$= 62500 \text{ N (upward)}$$

2. (a) $N_{AG} = 100 \text{ N}$, $a_A = 2 \text{ m/s}^2$

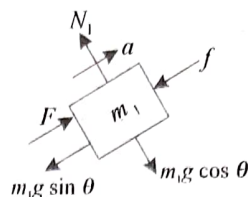
(b) $N_{BG} = 50 \text{ N}$, $N_{BG} = 150 \text{ N}$, $a_A = a_B = 0$

(c) $N_{AG} = 100 \text{ N}$, $N_{BG} = 50 \text{ N}$, $N_{AB} = 20 \text{ N}$, $a_A = a_B = 2$

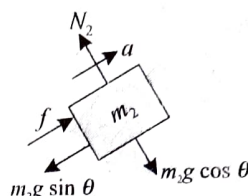
(d) $N_{BG} = 50 \text{ N}$, $N_{AG} = 150 \text{ N}$, $a_A = 2$, $a_B = 0$

3. (i) Let f be the normal reaction between the blocks.

FBD of mass m_1



FBD of mass m_2



(ii) From FBD of mass m_1 ;

$$F - (m_1 g \sin \theta + f) = m_1 a \quad (1)$$

From FBD of mass m_2 ;

$$f - m_2 g \sin \theta = m_2 a \quad (2)$$

Adding the equations (i) and (ii);

$$(m_1 + m_2)a = F - (m_1 + m_2)g \sin \theta$$

$$a = \frac{F - (m_1 + m_2)g \sin \theta}{(m_1 + m_2)}$$

(iii) From equation (2)

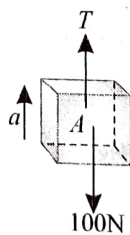
$$f = m_2 a + m_2 g \sin \theta$$

$$f = m_2 \left[\frac{F - (m_1 + m_2)g \sin \theta}{(m_1 + m_2)} \right] + m_2 g \sin \theta$$

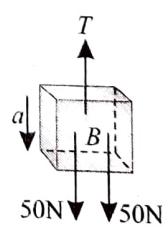
$$f = m_2 \left[\frac{F - (m_1 + m_2)g \sin \theta + (m_1 + m_2)g \sin \theta}{(m_1 + m_2)} \right]$$

$$f = \frac{m_2 F}{m_1 + m_2}$$

4.



F.B.D of A



F.B.D of B

Let acceleration of blocks is a .

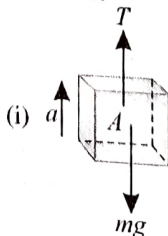
Equation of motion of 'A'

$$T - 100 = 10a$$

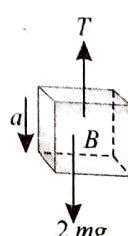
Equation of motion of 'B' $(50 + 50) - T = 5a$

From (i) and (ii), $a = 0$ and $T = 100 \text{ N}$

5. Method 1:



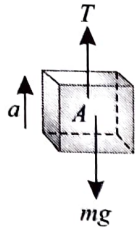
F.B.D of A



F.B.D of B

Equation of motion of 'A', $T - mg = ma$
 Equation of motion of 'B', $2mg - T = 2ma$
 From (i) and (ii), $a = \frac{g}{3}$

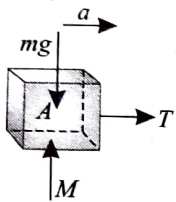
(ii)



F.B.D of A

Here $T = 2mg$
 Equation of motion of 'A', $T - mg = ma$
 From (i) and (ii), $a = g$

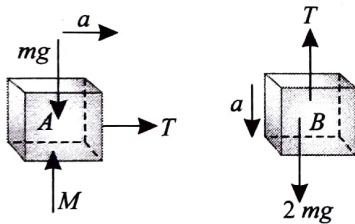
(iii)



F.B.D of A

Here $T = 2mg$
 Equation of motion of 'A', $T = ma$
 From (i) and (ii), $a = 2g$

(iv)



F.B.D of A

F.B.D of B

Equation of motion of 'A', $T = ma$
 Equation of motion of 'B', $2mg - T = 2ma$

From (i) and (ii), $a = \frac{2}{3}g$

Method 2: (i) $a = \frac{2mg - mg}{3m} = \frac{g}{3}$

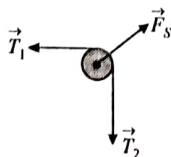
$$(ii) \quad a = \frac{2mg - mg}{m} = g$$

$$(iii) \quad a = \frac{2mg}{m} = 2g$$

$$(iv) \quad a = \frac{2g}{3}$$

6. The F.B.D. of pulley is as shown

Let $\vec{T}_1$, $\vec{T}_2$ and $\vec{F}_S$ be the forces exerted by the horizontal string, vertical string and the support on the massless pulley respectively.



(i)
 (ii)

Then

$$\vec{T}_1 + \vec{T}_2 + \vec{F}_S = 0$$

$$\text{or } |\vec{F}_S| = |\vec{T}_1 + \vec{T}_2| = 2\sqrt{2}mg$$

(Tension in each string is $|\vec{T}_1| = |\vec{T}_2| = 2mg$)

Single Correct Answer Type

7. (b) The sum of forces $\vec{F}_1 + \vec{F}_2 = m\vec{a}$

$$= 5(10 \cos \theta \hat{i} + 10 \sin \theta \hat{j}) = 30\hat{i} + 40\hat{j}$$

$$\therefore \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

$$\therefore \vec{F}_3 = -(\vec{F}_1 + \vec{F}_2) = -30\hat{i} - 40\hat{j}$$

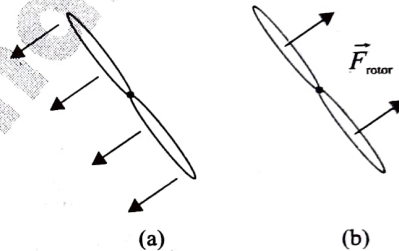
(i)
 (ii)

8. (a) Net force on the particle is zero so that $\vec{v}$ remains unchanged.

9. (c) $\vec{F}_{\text{gravitational}}$ acts downwards.

$\vec{F}_{\text{drag}}$ acts opposite to the direction of motion.

Now let us see how $\vec{F}_{\text{rotor}}$ acts.



Wings of helicopter will push the air perpendicular to their plane of rotation as shown in figure (a). From third law, air will apply force $\vec{F}_{\text{rotor}}$ on the wings as shown in figure (b).

There is no force ma . Basically ma is the resultant of all the forces acting on a body. moreover, here a is zero because velocity of helicopter is constant.

10. (c) Acceleration of all blocks in both cases is the same and is

$$a = \frac{F}{m + M}$$

In case I : required normal reaction R_I is equal to net force on block of mass m

$$\therefore R_I = ma$$

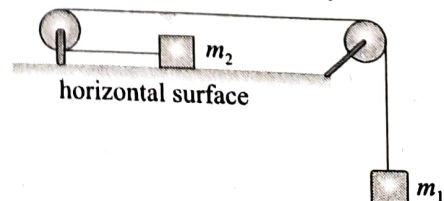
In case II : required normal reaction R_{II} is equal to net force on block of mass M .

$$\therefore R_{II} = Ma$$

Hence $R_{II} > R_I$

11. (d) The magnitude of acceleration of hanging 2 kg block is highest in figure B and least in figure A. The magnitude of weight acting downward on each hanging block (of same masses) is same. Hence tension shall be highest for block having least acceleration (because it is tension which decreases acceleration of each block). Therefore $T_A > T_C > T_B$.

12. (d) The acceleration of block of mass m_1 is



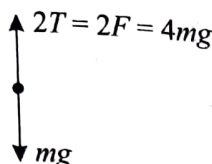
$$a = \frac{m_1}{m_1 + m_2} g = 5 \text{ m/s}^2$$

From FBD of block m_1

$$m_1 g - T = m_1 a$$

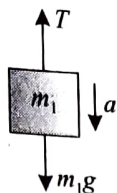
$$\Rightarrow T = m_1 (g - a) = 1 \times 5 = 5 \text{ N}$$

13. (d) F.B.D. of block



$$\Rightarrow 2T - mg = ma$$

$$a = 3g$$



2. (d) Range of resultant of F_1 and F_2 varies between $(3 + 5) = 8 \text{ N}$ and $(5 - 3) = 2 \text{ N}$. It means for some value of angle (θ), resultant 6 can be obtained. So, the resultant of 3 N, 5 N and 6 N may be zero and the forces may be in equilibrium.

3. (a) FBD of sphere:

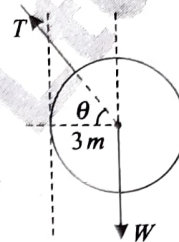
$$w = T \sin \theta$$

$$\text{and } \cos \theta = \frac{3}{5} \Rightarrow \theta = 53^\circ$$

$$\Rightarrow T = \frac{w}{\sin 53^\circ} = \frac{5w}{4}$$

4. (a) Resolving forces at point A along string AB

$$w_1 \cos 37^\circ = w_2$$



Multiple Correct Answers Type

14. (c, d)

Consider the F.B.D of balloon
Net force on balloon is zero. So

$$\vec{F}_3 + \vec{F}_4 = 0$$

$$-\vec{F}_1 - \vec{F}_2 = 0 \Rightarrow \vec{F}_1 + \vec{F}_2 = 0$$

$\vec{F}_1$ and $\vec{F}_2$ can't be action-reaction pair because they do not act between the same two bodies. Similarly for $\vec{F}_3$ and $\vec{F}_4$.

15. (a, b, d)

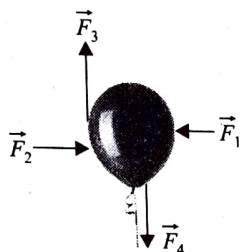
- (a) For the system of (horse + cart), only $\vec{F}_{c/h}$ and $\vec{F}_{h/c}$ will be internal forces. All other forces will be external.

- (b) As (horse + cart) system is moving with constant velocity, so net horizontal force on it should be zero.

$$\text{For this } \vec{F}_{h/g} + \vec{F}_{c/g} = 0$$

- (c) $\vec{N}_{C/g}$ and $\vec{F}_{C/E}$ can't be action-reaction pairs, as they act on same body.

- (d) $\vec{F}_{C/h}$ and $\vec{f}_{h/C}$ are action-reaction pairs.



5. (c) $T_1 \sin \theta_1 = 2mg$

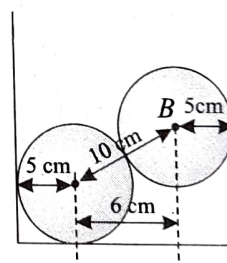
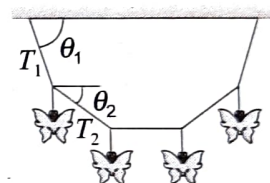
$$T_2 \sin \theta_2 = mg$$

$$T_1 \cos \theta_1 = T_2 \cos \theta_2$$

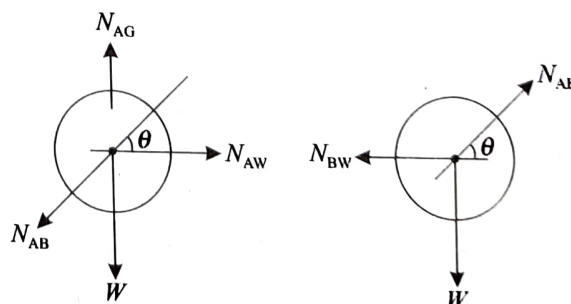
$$2mg \cot \theta_1 = mg \cot \theta_2$$

$$\Rightarrow \tan \theta_1 = 2 \tan \theta_2$$

6. (c)



The FBDs of A and B are:



For the horizontal equilibrium of the system (A and B):

$$N_{AW} = N_{BW}$$

and by diagrams:

$$\cos \theta = \frac{6}{10} \Rightarrow \theta = 53^\circ$$

Applying NLM on sphere B

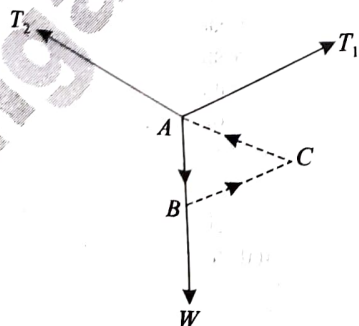
$$\Rightarrow N_{AB} \sin \theta = w$$

DPP 6.3

Single Correct Answer Type

1. (c) $\vec{AB} = \vec{W} \cdot \vec{BC} = \vec{T}_1 \cdot \vec{CA} = \vec{T}_2$

$$\vec{AB} = \vec{BC} = \vec{CA} = 0 \text{ (as the block is at rest)}$$



(1)

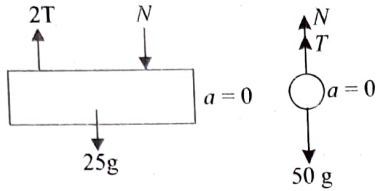
(2)

$$\text{and } N_{BW} = N_{AB} \cos \theta = \frac{w}{(4/5)} \times \frac{3}{5} = \frac{3w}{4}$$

$$\text{So } N_{AW} = N_{BW} = \frac{3w}{4}$$

7. (b)

F.B.D. of man and platform



$$2T + N = 25g$$

$$3T = 750$$

$$N + T = 50g$$

$$T = 250 \text{ N}$$

8. (d) In a position as shown

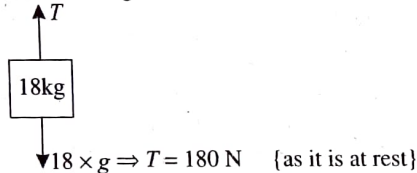
$$2T \cos \theta = W$$

$$T = \frac{W}{2 \cos \theta}$$

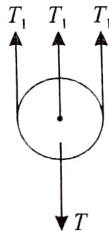
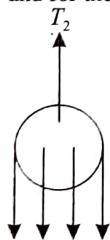
For AB to be horizontal, $\theta = 90^\circ$

$$T \rightarrow \infty$$

9. (d) FBD of 18 kg block



For the lower pulley:

Clearly: $3T_1 = T \Rightarrow T_1 = 60 \text{ N}$
and for the upper pulley:

$$T_1 \quad T_1 \quad T_1 \quad T_1 \quad T_2 = 4T_1 = 240 \text{ N}$$

where T_2 is the force exerted by the ceiling on the pulley.

Multiple Correct Answers Type

10. (a, c)

For painter;

$$R + T - mg = ma$$

$$R + T = m(g + a)$$

For the system;

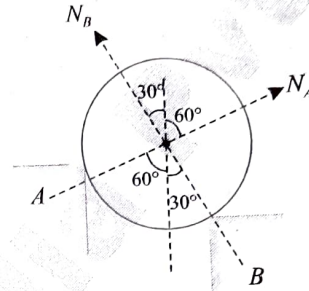
$$2T - (m + M)g = (m + M)a$$

$$2T = (m + M)(g + a) \quad (2)$$

where; $m = 100 \text{ kg}$
 $M = 50 \text{ kg}$
 $a = 5 \text{ m/sec}^2$
 $\therefore T = \frac{150 \times 15}{2} = 1125 \text{ N}$

and $R = 375 \text{ N}$

11. (b, c)

For equilibrium $N_A \cos 60^\circ + N_B \cos 30^\circ = Mg$
and $N_A \sin 60^\circ = N_B \sin 30^\circ$ 

$$\text{On solving } N_B = \sqrt{3}N_A; N_A = \frac{Mg}{2}$$

12. (a, c)

Maximum force, the rope B can withstand is 300 N, so the maximum tension which can be produced in rope A should be

$$= \frac{300}{2} = 150 \text{ N}$$

(i) For maximum acceleration:

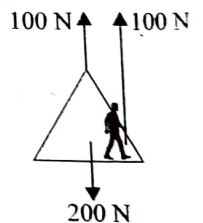
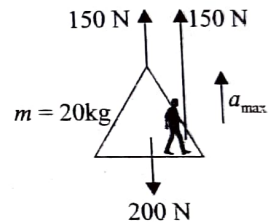
$$150 + 150 - 200 = 20a_{\max}$$

$$a_{\max} = 5 \text{ m/s}^2$$

(ii) Now to hoist up (say with uniform velocity), minimum tension in A should be 100 N

(or painter should apply force of 100 N on rope A), then minimum tension in B should be 200 N. So B should withstand minimum tension of 200 N. Same is the case to move down slowly.

(iii) When the painter is at rest the tension in A should be 100 N to balance the weight of painter and platform. So painter will exert 100 N on rope A.



DPP 6.4

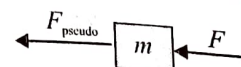
Subjective Type

1. In each case of pseudo force will act opposite to the direction of acceleration of the observer. It is equal to

$$\vec{F}_{\text{pseudo}} = -m\vec{a}$$

If a horizontal force starts acting on the block the FBD of block with respect to observer,

then acceleration of the block w.r.t observer



$$a_{m, \text{observer}} = \frac{F_{\text{pseudo}} + F}{m} = \frac{ma + F}{m} = a + \frac{F}{m}$$

$$\text{Hence } \vec{a}_{m, \text{observer}} = -\left(a + \frac{F}{m}\right)\hat{i}$$

2. The pseudo force is observed from accelerating observer and acts in the direction opposite to the direction of the observer and magnitude is equal to mass of the object multiple by the acceleration of the observer.

Hence $F_{\text{pseudo}} = ma$ (downward) because the direction of the acceleration on the lift is in upward direction.

If lift falls freely, the acceleration of the lift along with observer will be in downward direction and equal to acceleration due to gravity. So the direction of pseudo force will be in upward direction and magnitude is equal to

$$F_{\text{pseudo}} = mg \text{ (upward)}$$

3. The acceleration of the system (wedge + block)

$$a = \frac{F}{(M+m)} \text{ (rightward)}$$

The pseudo force on the block m will act in left direction

$$(\vec{F}_m)_{\text{pseudo}} = m\vec{a} = -m\left(\frac{F}{M+m}\right)(\hat{i})$$

The observer sitting on the wedge will also move with the acceleration of the system. Hence pseudo force on the wedge

$$(\vec{F}_M)_{\text{pseudo}} = M\vec{a} = -M\left(\frac{F}{M+m}\right)(\hat{i})$$

The pseudo force is not a real force so it will not have any action-reaction pairs.

4. Applying Newton's law on the system in horizontal direction $F = (M+m)a$.

Now consider the equilibrium of block B w.r.t. block M

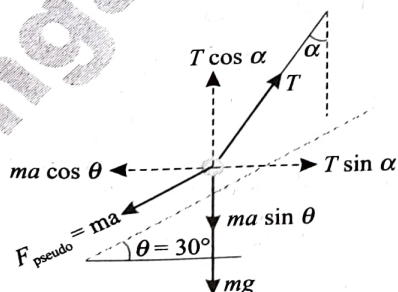
$$F^2 = (mg)^2 + (ma)^2$$

$$= (mg)^2 + \left(m \frac{F}{m+M}\right)^2$$

$$\therefore F^2 = \frac{m^2 g^2}{1 - \frac{m^2}{(m+M)^2}}; F = \frac{mg}{\sqrt{1 - \left(\frac{m}{m+M}\right)^2}}$$

Single Correct Answer Type

5. (b) In F.B.D of the bob resolving the components of pseudo force and tension in horizontal and vertical direction.



$$T \sin \alpha = ma \cos \theta$$

$$T \cos \alpha = ma \sin \alpha + mg$$

$$\text{From (i) and (ii), } \tan \alpha = \frac{a \sin \theta}{a \sin \theta + g}$$

$$\text{Here } a = 10 \text{ m/s}^2, \theta = 30^\circ$$

$$\text{After solving we get } \tan \alpha = \frac{1}{\sqrt{3}} \text{ or } \alpha = 30^\circ$$

6. (a) Acceleration of wedge = $g \sin \theta$

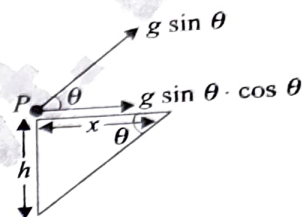
FBD of particle w.r.t. wedge:

Displacement of particle on wedge

$$x = \left(\frac{h}{\tan \theta}\right)$$

Acceleration of particle along displacement = $g \sin \theta \cos \theta$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2h/\tan \theta}{g \sin \theta \cos \theta}} = \sqrt{\frac{2h}{g \sin^2 \theta}}$$

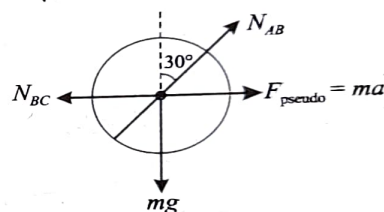


7. (c) The free body diagram of cylinder w.r.t carriage is as shown. Since net acceleration of cylinder is horizontal.

$$N_{AB} \cos 30^\circ = mg$$

$$\text{or } N_{AB} = \frac{2}{\sqrt{3}} mg$$

(1)



F.B.D of cylinder w.r.t to carriage

$$\text{and } N_{BC} = ma + N_{AB} \sin 30^\circ$$

(2)

Hence N_{AB} remains constant and N_{BC} increases with increase in a .

8. (a) The situation is shown in figure.

When the car moves towards right with acceleration a the rope carrying the mass makes an angle θ .

Forces acting on the body are:

(1) Weight mg downwards.

(2) Tension T in the rope.

(3) Pseudo force ma towards left.

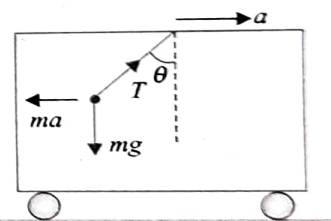
Since the body is in equilibrium under the action of three forces, we can apply condition of equilibrium.

$$T \cos \theta - mg = 0; T \sin \theta = ma$$

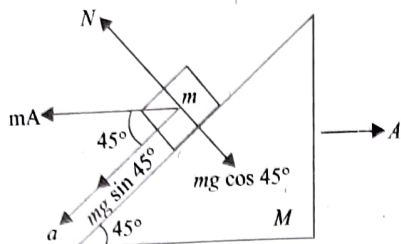
$$\tan \theta = \frac{a}{g} \text{ or } a = g \tan \theta$$

$$\text{Acceleration, } a = \frac{1}{3} \left(5 - \frac{5}{3}\right) = \frac{10}{9} \text{ m/sec}^2$$

$$\frac{10}{9} = 10 \tan \theta \text{ or } \tan \theta = \frac{1}{9} \Rightarrow \theta = \tan^{-1} \left(\frac{1}{9}\right)$$



9. (c) a is acceleration of block w.r.t. wedge



For block:

$$N + mA \sin 45^\circ = mg \cos 45^\circ$$

For wedge: $N \sin 45^\circ = MA$

From (i) and (ii) $A = 1 \text{ m/s}^2$

10. (c) Acceleration along the plane with respect to lift is,

$$a = (a_0 + g) \sin \theta$$

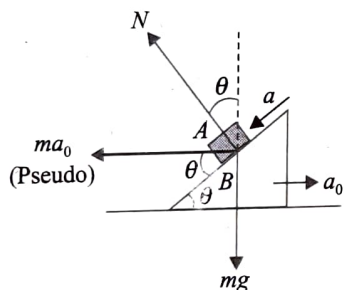
Initial velocity = 0

$$v^2 = u^2 + 2as$$

$$v = \sqrt{2(a_0 + g) \sin \theta \frac{L}{\cos \theta}}$$

Multiple Correct Answers Type

11. (a, d)



$$ma_0 \sin \theta + N = mg \cos \theta$$

$$\Rightarrow mg \cos \theta$$

$$\Rightarrow N = mg \cos \theta - ma_0 \sin \theta$$

$$\Rightarrow N < mg \cos \theta$$

Hence, (D) is true.

$$ma_0 \cos \theta + mg \sin \theta = ma$$

$$\Rightarrow a = g \sin \theta + a_0 \cos \theta$$

Hence, acceleration of A

$$= \sqrt{(a - a_0 \cos \theta)^2 + (a_0 \sin \theta)^2} > g \sin \theta$$

Comprehension Type

12. (a) Given $a = g \tan \theta$

Acceleration of block relative to wedge is zero.

Velocity of block = v

Distance = l

$$\therefore \text{Time } T = \frac{l}{v}$$

13. (b) Normal reaction $N_R = mg \cos \theta + ma \sin \theta$

$$N_R = mg \cos \theta + mg \frac{\sin^2 \theta}{\cos \theta} = mg \sec \theta$$

14. (d) $a = g \tan \theta$

$$\frac{F}{(m+M)} = g \tan \theta$$

$$F = (m+M)g \tan \theta$$

15. (a) Since block is moving with zero acceleration relative to wedge

$\Rightarrow$ reaction force = mg

$$\therefore \text{Normal reaction offered by ground } N_G = Mg + mg = (M+m)g$$

DPP 6.5

Subjective Type

1. $l_1 + l_2 + l_3 = \text{constant}$

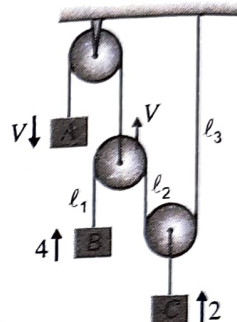
$$\dot{l}_1 + \dot{l}_2 + \dot{l}_3 = 0$$

$$(V-4) + (V-2) + (-2) = 0$$

$$\Rightarrow 2V = 8 \Rightarrow V = 4 \text{ m/s} \downarrow$$

$$2. u = \frac{0+v_1}{2}, \frac{v_1+v_2}{2} = v, \frac{-v+u}{2} = v$$

$$\text{Hence } v = \text{velocity of } M = \frac{3u}{4}$$



Single Correct Answer Type

3. (b) Let $AB = l, B = (x, y)$

$$\vec{v}_B = v_x \hat{i} + v_y \hat{j}$$

$$\vec{v}_B = \sqrt{3} \hat{i} + v_y \hat{j}$$

$$x^2 + y^2 = l^2$$

$$2x v_x = 2y v_y = 0$$

$$\Rightarrow \sqrt{3} + \frac{y}{x} v_y = 0$$

$$\Rightarrow \sqrt{3} + (\tan 60^\circ) v_y = 0$$

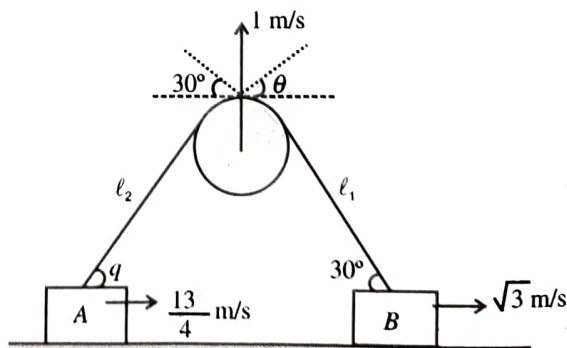
$$\Rightarrow v_y = -1$$

Hence from (i)

$$\vec{v}_B = \sqrt{3} \hat{i} - \hat{j}$$

Hence $v_B = 2 \text{ m/s}$

4. (a)



$$l_1 + l_2 = \text{constant}$$

$$\Rightarrow \frac{dl_1}{dt} + \frac{dl_2}{dt} = 0$$

$$\Rightarrow (\sqrt{3} \cos 30^\circ + 1 \cos 60^\circ) + 1 \sin \theta - \frac{13}{4} \cos \theta = 0$$

$$\Rightarrow 4 \sin \theta - 13 \cos \theta = -8$$

$$\Rightarrow 13 \cos \theta - 4 \sin \theta = 8.$$

Checking the options; we get $\theta = 37^\circ = \tan^{-1}\left(\frac{3}{4}\right)$.

5. (a) Let the magnitude of velocity of top of the rod be v_R towards left and the speed of block B be v .

$$l_1 + l_2 = \text{constant}$$

$$\therefore \frac{d\ell_1}{dt} + \frac{d\ell_2}{dt} = 0$$

$$\Rightarrow u - v_R \cos 60^\circ = 0 \quad (1)$$

$$l_3 + l_4 = \text{constant}$$

$$\therefore \frac{d\ell_3}{dt} + \frac{d\ell_4}{dt} = 0$$

$$\Rightarrow v_R \cos 30^\circ - v = 0$$

(2)

Solving (1) and (2) $v = \sqrt{3}u$

6. (a) In the frame of the lift first pulley will be stationary so velocity of second pulley will be zero and so in the frame of ground it moves with 5 m/s.

7. (d) For block A

$$mg - 2T = ma$$

For block B

$$T - mg = mb$$

and by constraint motion

$$(a - b) + a = 0$$

Solving (1), (2) and (3) get

$$a = -g/5$$

and $b = -2g/5$

8. (a) Let $a = \text{acc}^n$ of m_1 acc^n of pulley $= \frac{a+0}{2} = \frac{a}{2}$

If acceleration of $m_2 = b$

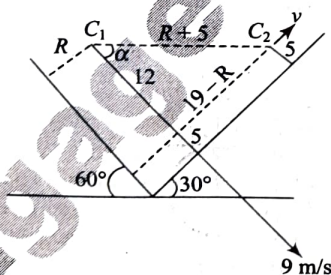
$$\text{Then } 0 + \frac{b}{2} = \frac{a}{2}$$

Hence $a = b$

$$T = m_1 a, m_2 g - T = m_2 a$$

$$\therefore a = \frac{m_2 g}{m_1 + m_2}$$

9. (b)



$$9 \cos \alpha = v \sin \alpha \rightarrow$$

$$\frac{19 - R}{12} = \tan \alpha$$

$$(R + 5)^2 = (12)^2 + (19 - R)^2$$

$$\Rightarrow R = 10$$

Hence from (i) and (ii)

$$v = 12 \text{ m/s}$$

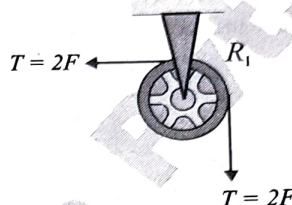
10. (c) F.B.D of the mass M :

$$T = F$$

$$ma = 2T = 2F \Rightarrow a = \frac{2F}{m}$$

So, acceleration of point P is $2a = \frac{4F}{m}$ (From constraint relation)

11. (c) Forces on the rod R_1 are shown in the figure:

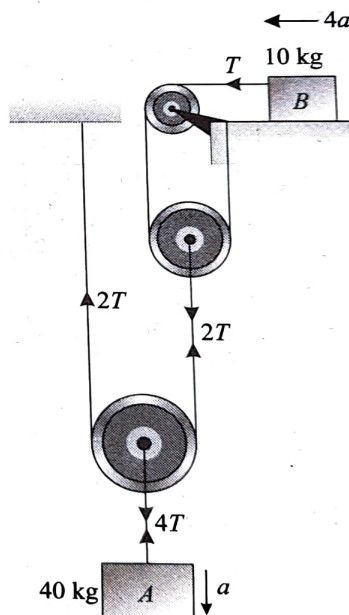


$$F_{\text{Net}} = 2\sqrt{2} F$$

Multiple Correct Answers Type

12. (b, d)

Applying NLM on 40 kg block



$$400 - 4T = 40 a$$

$$\text{For 10 kg block } T = 10.4 a$$

$$\text{Solving } a = 2 \text{ m/s}^2$$

$$T = 80 \text{ N}$$

(i)

(ii)

DPP 6.6

Single Correct Answer Type

1. (c) The F.B.D. of section of rope between A and B having acceleration a towards left is

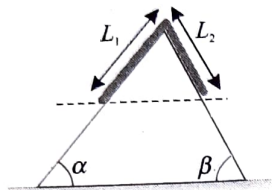


Applying Newton's second law on section AB and section AC of rope we get

$$T_B - T_A = Ma \text{ and } T_C - T_A = \frac{M}{5}a$$

$$\text{Solving } T_C = \frac{T_B + 4T_A}{5}$$

2. (a) Let L_1 and L_2 be the portions (of length) of rope on



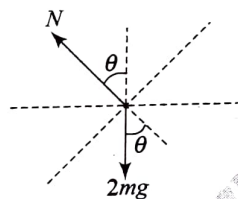
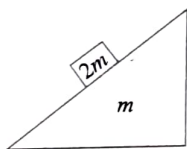
left and right surface of wedge as shown.

$\therefore$ Magnitude of acceleration of rope

$$a = \frac{\frac{M}{L}[L_1 \sin \alpha - L_2 \sin \beta]g}{M} = 0$$

$$(\because L_1 \sin \alpha = L_2 \sin \beta)$$

3. (a) FBD of block A



$$N \cos \theta = 2mg$$

[For accelerations of both the block A and wedge B be same, they will move along the horizontal]

$$N \sin \theta = ma$$

$$\cot \theta = \frac{2g}{a}$$

$$a = 2g \tan \theta$$

From (ii)

$$N = \frac{m}{\sin \theta} \cdot 2 \tan \theta \Rightarrow N = \frac{2mg}{\cos \theta}$$

4. (b) For block to move up



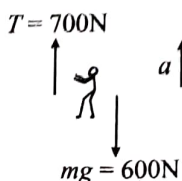
$$ma \cos \theta > mg \sin \theta$$

$$a > g \tan \theta$$

5. (d) For student A to just lift off the floor, tension T in string must be greater than or equal to 700 N.

The F.B.D. of student B is

Applying Newton's second law



$$T - mg = ma \Rightarrow 700 - 600 = 6a$$

$$\text{or } a = \frac{5}{3} \text{ m/s}^2$$

6. (a) The magnitude of the force (from the string) is $T = 30 \text{ N}$
 The x-component $= T \sin \theta = 30 \times 3/5 = 18 \text{ N}$
 The y-component $= T \cos \theta = 30 \times 4/5 = 24 \text{ N}$
 The total force on the block is:
 the x-component $= 18 \text{ N}$
 the y-component $= 24 - mg = 24 - 20 = 4 \text{ N}$
 The x-component of the acceleration $= 18 \text{ N} / 2 \text{ kg} = 9 \text{ m/s}^2$
 The y-component of the acceleration $= 4 \text{ N} / 2 \text{ kg} = 2 \text{ m/s}^2$

Multiple Correct Answers Type

7. (b, d)

Let a be acceleration of system and T be tension in the string.

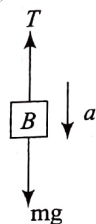
F.B.D of block A

$$mg \sin 30^\circ + T = ma$$

$$\frac{mg}{2} + T = ma$$

F.B.D of block B

$$mg - T = ma$$



Adding equation (i) and (ii); we get

$$2ma = \frac{3mg}{2} \Rightarrow a = \frac{3}{4}g$$

8. (a, b, c, d)

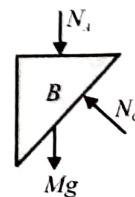
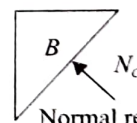
There is no horizontal force on block A, therefore it does not move in x-direction, whereas there is net downward force ($mg - N$) is acting on it, making its acceleration along negative y-direction.

Block B moves downward as well as in negative x-direction. Downward acceleration of A and B will be equal due to constrain, thus w.r.t. B, A moves in positive x-direction.

Due to the component of normal exerted by C on B, it moves in negative x-direction.

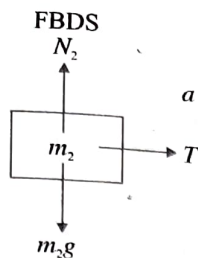
The force acting vertically downward on block B are mg and N_A (normal reaction due to block A). Hence the component of net force on block B along the inclined surface of B is greater than $mg \sin \theta$. Therefore the acceleration of 'B' relative to ground directed along the inclined surface of 'C' is greater than $g \sin \theta$.

$$T = \frac{mg}{4}$$

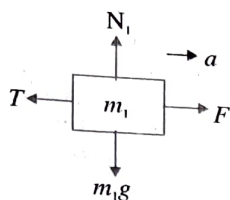
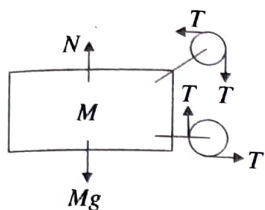


Matching Column Type

9. (a → q) (b → q) (c → r) (d → s)



$$T = m_2 a$$



$$F - T = m_1 a$$

$$F = (m_1 + m_2) a$$

$$T = m_2 a$$

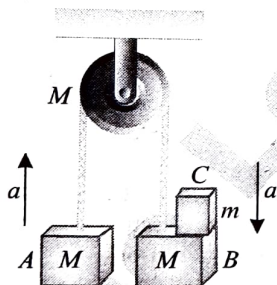
$$\Rightarrow a = \frac{F}{m_1 + m_2}$$

$$\therefore T = \frac{m_2 F}{m_1 + m_2} \quad F_x = 0, a_M = 0$$

Comprehension Type

Solution for Problems 10–12

10. (a) 11. (b) 12. (d)



By newtons law on system of (A, B, C)

$$10. (a) (M + m - M) g = (2M + m) a$$

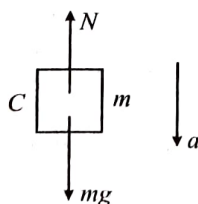
$$\therefore a = \frac{mg}{2M + m}$$

11. (b) Free body diagram 'C' block

$$mg - N = ma$$

$$\therefore N = m \left(g - \frac{gm}{2M + m} \right)$$

$$N = \frac{2M mg}{2M + m}$$



$$12. (d) T - mg = M \frac{mg}{2M + m} \text{ for A block}$$

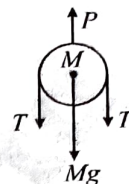
$$\therefore T = Mg + \frac{Mmg}{2M + m}$$

for pulley

$$P = 2T + Mg$$

$$= 2Mg + \frac{2Mmg}{2M + m} + Mg = \frac{6M + 3m + 2m}{2M + m} Mg$$

$$P = \left(\frac{6M + 5m}{2M + m} \right) Mg$$



DPP 6.7

Subjective Type

$$1. 40 = L + \frac{100}{K} \Rightarrow 60 = L + \frac{200}{K}$$

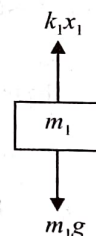
$$\therefore K = 5 \text{ and } L = 20$$

$$\text{Now } 30 = 20 + \frac{x}{5} \Rightarrow x = 50$$

2. Let the initial extension in top spring be x_1 and the compression in the bottom spring be x_3 respectively.For equilibrium of m_1

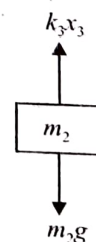
$$k_1 x_1 = m_1 g$$

(1)

For equilibrium of m_2

$$k_3 x_3 = m_2 g$$

(2)

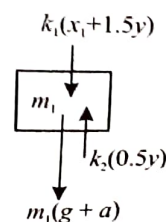
Let the elevator starts accelerating with an acceleration a in the upward direction. Now in the position of final equilibrium, let further compression in the bottom spring be y . $\therefore$ Further extension in the top spring = $1.5y$ and compression in the middle spring = $0.5y$

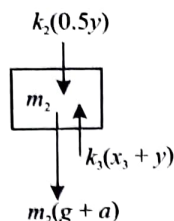
In the frame of lift

$$(a) m_1(g + a) = k_1(x_1 + 1.5y) + k_2(0.5y)$$

$$\therefore m_1 g = k_1 x_1$$

$$\therefore m_1 a = y \left(\frac{3}{2} k_1 + \frac{1}{2} k_2 \right) \quad (3)$$





$$m_2(g + a) = k_3(x_3 + y) - k_2(0.5y)$$

$$\therefore m_2g = k_3x_3$$

$$\therefore m_2a = y \left(k_3 - \frac{k_2}{2} \right) \quad (4)$$

(3) (4), we get

$$\frac{m_1}{m_2} = \frac{3k_1 + k_2}{2k_3 - k_2}$$

$$(b) \quad k_1 = k_2 = k_3 = 500 \text{ N/m}$$

$$m_1 = 2 \text{ kg}$$

$$a = 2.5 \text{ m/s}^2$$

using (3)

$$y = \frac{2 \times 2.5}{2 \times 500} = \frac{1}{200} \text{ m} = 0.5 \text{ cm}$$

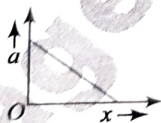
Tension in the middle spring = $k(0.5y)$

$$= 500 \times \frac{1}{2} \times \frac{1}{2} \times 10^{-2}$$

$$T = 1.25 \text{ N}$$

Single Correct Answer Type

3. (c) 5 N force will not produce any tension in spring without support of other 5 N force. So here the tension in the spring will be 5 N only.
4. (d) The fictitious force will act downwards. So the reading of spring balance will increase. In case of physical balance, the fictitious force will act on both the pans, so the equilibrium is not affected.
5. (b) As the mass of 10 kg has acceleration 12 m/s^2 therefore it apply 120 N force on mass 20 kg in a backward direction.
 $\therefore$ Net forward force on 20 kg mass = $200 - 120 = 80 \text{ N}$
6. (c) Let the initial compression of spring be l . Then the acceleration after the block travels a distance x is



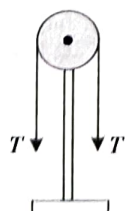
$$a = \frac{k}{m}(l - x)$$

$\therefore$ The graph of a vs. x is

7. (b) Reading of the weighing machine = $2T$ + weight of the machine.

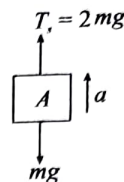
As weight of the machine is constant.

$$T = \frac{2m_1m_2}{m_1 + m_2}$$



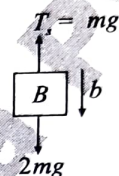
So reading is maximum for the case m_1m_2 is maximum as $m_1 + m_2$ in all cases is same.

8. (c) For first case tension in spring will be $T_s = 2mg$ just after 'A' is released.



$$2mg - mg = ma \Rightarrow a = g$$

In second case $T_s = mg$



$$2mg - mg = 2mb$$

$$b = g/2$$

$$a/b = 2$$

9. (b) If m_1 and m_2 are masses of blocks, then tension T in the string as well as spring is

$$T = \frac{2m_1m_2}{m_1 + m_2}g$$

$$T_1 = 2 \text{ g}, T_2 = 2.4 \text{ g and } T_3 = 1.33 \text{ g}$$

$$\therefore T_2 > T_1 > T_3$$

$$\text{or } x_2 > x_1 > x_3$$

10. (b) If reading of spring balance is T , then applying NLM on (man + ladder) system

$$T - (25 + 5)g = 25a$$

$$T - 30g = 25a \Rightarrow T - 300 = 25(1)$$

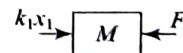
$$\Rightarrow T = 325 \text{ N} = 32.5 \text{ kg}$$

Multiple Correct Answers Type

11. (a, c, d)

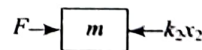
Let F be the force exerted by mass m on mass M .

FBD of mass M



$$F = k_1x_1 = 2 \times 3 = 6 \text{ N}$$

FBD of mass m



$$k_2x_2 = F = 6 \text{ N to the left}$$

Hence the force exerted on block of mass m by the right spring (k_2x_2) is 6 N to the left. From FBD, the normal reaction (F) between blocks is 6 N.

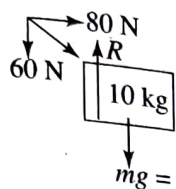
As system is at rest, net force on block of mass m is zero.

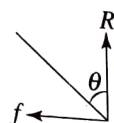
CHAPTER 7

Solutions S.51

DPP 7.1

Subjective Type

1. 
 $R = mg + 60 = 160 \text{ N}$
 $f = 80 \text{ N} (\because \text{No sliding})$
 angle of friction $\theta = \tan^{-1} \frac{f}{R} = \tan^{-1} \frac{80}{160}$
 $\theta = \tan^{-1} \frac{1}{2}$



2. The block begins to slide if

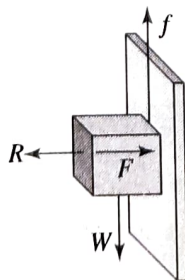
$$F \cos 37^\circ = \mu (mg - F \sin 37^\circ)$$

$$5t [\cos 37^\circ + \mu \sin 37^\circ] = \mu mg$$

$$5t \left[\frac{4}{5} + \frac{3}{5} \right] = 70 \quad \text{or} \quad t = 10 \text{ second}$$

Single Correct Answer Type

3. (a) Applied force = 2.5 N
 Limiting friction = $\mu mg = 0.4 \times 2 \times 9.8 = 7.84 \text{ N}$
 For the given condition applied force is smaller than limiting friction.
 $\therefore$ Static friction on a body = Applied force = 2.5 N
4. (c) The person applying more frictional force on ground will win.
5. (d) $N = mg = 40 \text{ N}$
 $(f_s)_{\max} = \mu N = (0.8) (40) = 32 \text{ N}$
 $f_s = \text{ext. force} = 30 \text{ N}$
 $R^2 = N^2 + f_s^2 = (50)^2$
 $\therefore R = 50 \text{ N}$
6. (c) Here applied horizontal force F acts as normal reaction.
 For holding the block
 Force of friction = Weight of block
 $f = W \Rightarrow \mu R = W \Rightarrow \mu F = W$
 $\Rightarrow F = \frac{W}{\mu}$
 As $\mu < 1 \therefore F > W$



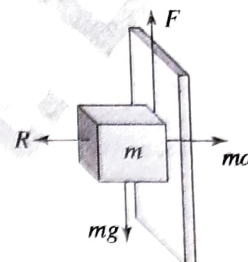
7. (a) For the limiting condition upward friction force between board and block will balance the weight of the block.

$$\text{i.e., } F > mg$$

$$\Rightarrow \mu(R) > mg$$

$$\Rightarrow \mu(ma) > mg$$

$$\Rightarrow \mu > \frac{g}{a}$$

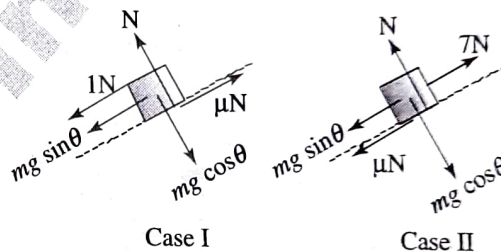


8. (c) For the sliding not to occur when $\tan \theta \leq \mu$

$$\tan \theta = \frac{dy}{dx} = \frac{2x}{a} = \frac{2\sqrt{y}}{\sqrt{a}} = 2\sqrt{\frac{y}{a}}$$

$$\therefore 2\sqrt{\frac{y}{a}} \leq \mu \quad \text{or} \quad y \leq \frac{a\mu^2}{4}$$

9. (d)



Case I: When block is about to slide down

$$1 + mg \sin \theta = \mu mg \cos \theta \quad (1)$$

Case II: When block is about to slide up

$$mg \sin \theta + \mu mg \cos \theta = 7 \quad (2)$$

$$\text{From (1) and (2) } \mu = \frac{4}{3\sqrt{3}}$$

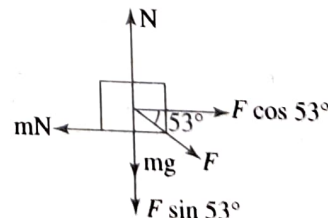
10. (c) Max. frictional force

$$f_{\max} = \mu N$$

$$= \mu (mg + F \sin 53^\circ)$$

$$= 0.2 \left(20 \times 10 + 30 \times \frac{4}{5} \right)$$

$$= 44.8 \text{ N}$$



As applied horizontal force is $F \cos 53^\circ$

$$= 18 \text{ N} < f_{\max}, \text{ friction force will also be } 18 \text{ N}$$

Multiple Correct Answers Type

11. (a, c)

In cycling, the rear wheel moves by the force communicated to it by pedalling while front wheel moves by itself. So, while pedalling a bicycle, the force exerted by rear wheel on ground makes force of friction act on it in the forward direction (like walking). Front wheel moving by itself experience force of friction in backward direction (like rolling of a ball). [However, if pedalling is stopped both wheels move by themselves and so experience force of friction in backward direction].

12. (b, c, d)

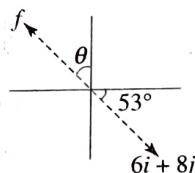
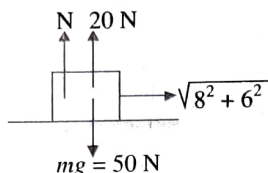
$$N = 50 - 20 = 30 \text{ N}$$

Limiting friction force = $\mu N = 12 \text{ N}$ and applied force in horizontal direction is less than the limiting friction force, therefore the block will not slide.

For equilibrium in horizontal direction, friction force must be equal to 10 N .

From the top view, it is clear that $\theta = 37^\circ$ i.e. 127° from the x -axis that is the direction of the friction force. It is opposite to the applied force.

$$\text{Contact force} = \sqrt{N^2 + f^2} = 10\sqrt{10} \text{ N}$$



DPP 7.2

Single Correct Answer Type

1. (c) Let the body is acted upon by a force at an angle θ with horizontal.

FBD:

$$F \cos \theta = \mu(mg - F \sin \theta)$$

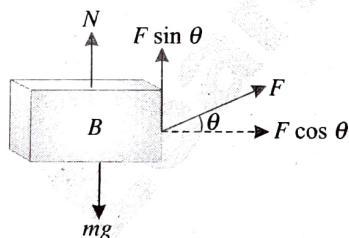
$$\Rightarrow F = \frac{\mu mg}{\cos \theta + \mu \sin \theta}$$

For min. force: $(\cos \theta + \mu \sin \theta)$ should be max.

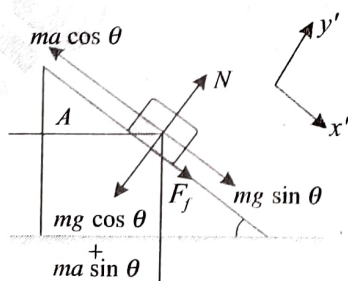
$$\Rightarrow -\sin \theta + \mu \cos \theta = 0$$

$$\Rightarrow \tan \theta = \mu \text{ or } \theta = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = 30^\circ \text{ Substituting}$$

$$F_{\min} = 12.5 \text{ kgf}$$



2. (b) FBD of block B w.r.t. wedge A, for maximum 'a':



Perpendicular to wedge:

$$\Sigma f_y' = (mg \cos \theta + ma \sin \theta - N) = 0$$

$$\text{and } \Sigma f_x' = mg \sin \theta + \mu N - ma \cos \theta = 0 \text{ (for maximum } a)$$

$$\Rightarrow mg \sin \theta + \mu(mg \cos \theta + ma \sin \theta) - ma \cos \theta = 0$$

$$\Rightarrow a = \frac{(g \sin \theta + \mu g \cos \theta)}{\cos \theta - \mu \sin \theta}$$

For $\theta = 45^\circ$

$$a = g \left(\frac{\tan 45^\circ + \mu}{\cot 45^\circ - \mu} \right); a = g \left(\frac{1 + \mu}{1 - \mu} \right)$$

3. (d) Limiting friction

$$= \mu_s R = \mu_s mg = 0.5 \times 60 \times 10 = 300 \text{ N}$$

Kinetic friction

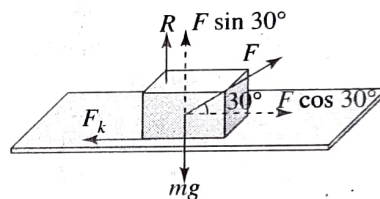
$$= \mu_k R = \mu_k mg = 0.4 \times 60 \times 10 = 240 \text{ N}$$

Force applied on the body = 300 N and if the body is moving then, net accelerating force

$$= \text{Applied force} - \text{Kinetic friction}$$

$$\Rightarrow ma = 300 - 240 = 60 \quad \therefore a = \frac{60}{60} = 1 \text{ m/s}^2$$

4. (a)



$$\text{Kinetic friction} = \mu_k R = 0.2(mg - F \sin 30^\circ)$$

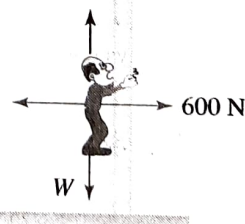
$$= 0.2 \left(5 \times 10 - 40 \times \frac{1}{2} \right) = 0.2(50 - 20) = 6 \text{ N}$$

Acceleration of the block

$$= \frac{F \cos 30^\circ - \text{Kinetic friction}}{\text{Mass}}$$

$$= \frac{40 \times \frac{\sqrt{3}}{2} - 6}{5} = 5.73 \text{ m/s}^2$$

$$5. (d) \text{ Net downward acceleration} = \frac{\text{Weight} - \text{Friction force}}{\text{Mass}}$$



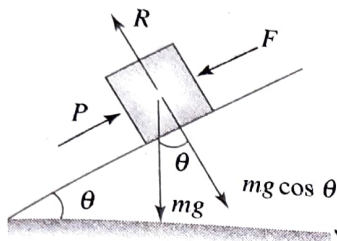
$$= \frac{(mg - \mu R)}{m} = \frac{60 \times 10 - 0.5 \times 600}{60}$$

$$= \frac{300}{60} = 5 \text{ m/s}^2$$

6. (a) Retardation in upward motion $= g(\sin \theta + \mu \cos \theta)$
 $\therefore$ Force required just to move up $F_{up} = mg(\sin \theta + \mu \cos \theta)$
 Similarly for downward motion $a = g(\sin \theta + \mu \cos \theta)$
 $\therefore$ Force required just to prevent the body sliding down
 $F_{db} = mg(\sin \theta - \mu \cos \theta)$

According to problem $F_{up} = 2F_{db}$
 $\Rightarrow mg(\sin \theta + \mu \cos \theta) = 2mg(\sin \theta - \mu \cos \theta)$
 $\Rightarrow \sin \theta + \mu \cos \theta = 2\sin \theta - 2\mu \cos \theta$
 $\Rightarrow 3\mu \cos \theta = \sin \theta \Rightarrow \tan \theta = 3\mu$
 $\Rightarrow \theta = \tan^{-1}(3\mu) = \tan^{-1}(3 \times 0.25) = \tan^{-1}(0.75)$

7. (d)



Net force along the plane

$$= P - mg \sin \theta = 750 - 500 = 250 \text{ N}$$

$$\text{Limiting friction} = F_1 = \mu_s R = \mu_s mg \cos \theta$$

$$= 0.4 \times 102 \times 9.8 \times \cos 30 = 346 \text{ N}$$

As net external force is less than limiting friction, therefore friction on the body will be 250 N.

8. (c) The tension in the string initially is zero. If $\mu_1 > \tan \alpha$ and $\mu_2 > \tan \beta$, both the blocks will not move down the incline and the tension in the string shall continue to remain zero.

9. (c) The free body diagram of the block is as shown in the figure.

N is the normal reaction exerted by wedge on the block.

The wedge moves towards left with acceleration ' a ', then the component of acceleration of block normal to the plane is ' $a \sin \theta$ '.

Applying Newton's second law to the block normal to plane.

$$mg \cos \theta - N = ma \sin \theta$$

For N to be zero $a = g \cot \theta$

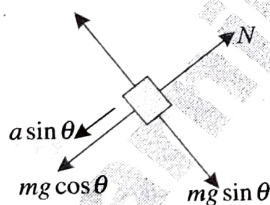
Hence the friction shall be zero when $a = g \cot \theta$.

10. (d) Let the value of ' a ' be increased from zero. As long as $a \leq \mu g$, there shall be no relative motion between m_1 or m_2 and platform, that is, m_1 and m_2 shall move with acceleration a .

As $a > \mu g$ the acceleration of m_1 and m_2 shall become μg each.

Hence at all instants the velocity of m_1 and m_2 shall be same

$\therefore$ The spring shall always remain in natural length.



$$mg(1 - \sin \theta) = \mu mg \cos \theta$$

$$\mu = \sec \theta - \tan \theta$$

So, choice (b) is correct.

- (d) The block m_1 will just begin to move down the plane, if the downward force ($m_1 g \sin \theta - \mu m_1 g \cos \theta$) on m_1 just equals the upward force m_2 of acting on m_1 due to m_2 if

$$m_2 g = m_1 g (\sin \theta - \mu \cos \theta)$$

$$m_2 g = 2m_2 g (\sin \theta - \mu \cos \theta)$$

$$\frac{1}{2} = \sin \theta - \mu \cos \theta$$

$$\frac{1}{2 \cos \theta} = \tan \theta - \mu$$

$$\mu = \tan \theta - \frac{1}{2} \sec \theta$$

12. (a, c)

$$mg \sin \theta + mg \cos \theta = ma$$

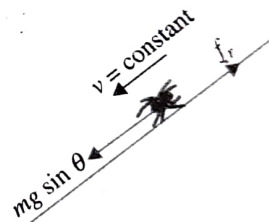
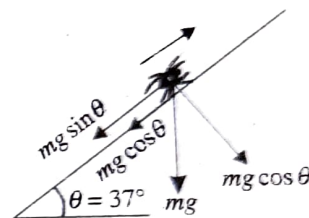
$$= 10 \left[\frac{3}{5} + \frac{4}{5} \right] = 14 \text{ m/sec}^2$$

$$\text{If } f_r = mg \sin \theta$$

$$= mg \times \frac{3}{5} = \frac{3mg}{5} = f_{r \max}$$

$$f_r < f_{r \max}$$

$$\frac{3mg}{5} < \frac{4mg}{5} \text{ hence insect can move with constant velocity.}$$



Comprehension Type

13. (c) Angle (θ) of repose;

$$m(g + a) \sin \theta = f$$

$$m(g + a) \cos \theta = R$$

$$\therefore \frac{f}{R} = \tan \theta$$

$$\theta = \tan^{-1} \left(\frac{f}{R} \right) = \alpha$$

Hence angle of repose does not change.

14. (a) To slide $mg \sin \theta > \mu mg \cos \theta$

$$\sin \theta > \mu \cos \theta$$

$$\theta > \tan^{-1} \mu$$

15. (b) Shear force $= \mu mg \cos \theta$

$$= \frac{3}{4} mg \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{8} mg = 0.06 mg$$

Multiple Correct Answers Type

11. (b, d)

- (b) The blocks m_1 will just begin to move up the plane if the downward force $m_2 g$ due to mass m_2 trying to pull the mass m_1 up the plane just equals the force

$$m_2 g = (m_1 g \sin \theta + \mu m_1 g \cos \theta)$$

$$mg = mg \sin \theta + \mu mg \cos \theta$$

But, pulling force

$$= mg \sin 30^\circ = 0.5 mg < f_{\max}$$

$\therefore$ block does not slide.

Hence frictional force (shear force) between the block of the plane at this situation will be

$$mg \sin 30^\circ = \frac{mg}{2} \left(\text{not } \frac{3\sqrt{3}}{8} mg \right)$$

Alternate Sol.

$$\tan \theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{2} = \frac{1.73}{3} = 0.58 < \mu$$

$\therefore$ block does not slide.

$$\therefore f_s = mg \sin 30^\circ$$

DPP 7.3

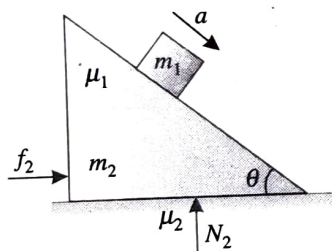
Subjective Type

1. Taking block + wedge as system and applying NLM in horizontal direction

$$f_2 = m_1 a \cos \theta$$

$$m_1 [g(\sin \theta - \mu_1 \cos \theta)] \cos \theta$$

Again applying NLM in vertical direction



$$(m_1 + m_2)g - N_2 = m_1 a \sin \theta$$

$$N_2 = (m_1 + m_2)g - m_1 \sin \theta (g \sin \theta - \mu_1 g \cos \theta)$$

For limiting condition

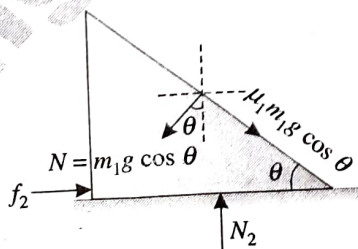
$$f_2 = \mu_2 N_2$$

From (1) and (2)

$$\mu_2 = \frac{m_1 \cos \theta (g \sin \theta - \mu_1 g \cos \theta)}{(m_2 + m_2)g - m_1 \sin \theta (g \sin \theta - \mu_1 g \cos \theta)}$$

$$\text{Using values } \mu_2 = \frac{1}{8}$$

Alternate Solution:



Applying NLM in vertical direction

$$N_2 = m_2 g + m_1 g \cos^2 \theta + \mu_1 m_1 g \sin \theta \quad (1)$$

Applying NLM in horizontal direction

$$f_2 = \mu_2 N = m_1 g \cos \theta + \sin \theta - \mu_1 m_1 g \cos^2 \theta$$

$$\text{On Solving } \mu_2 = \frac{1}{8}$$

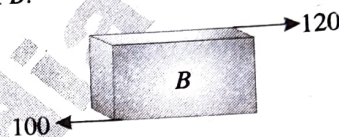
Single Correct Answer Type

2. (b) The friction force acting between the man and the block is:

$$F_f = 60 \times 20 = 120 \text{ N}$$

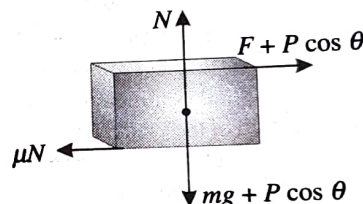
$$\text{and not } 0.3 \times 60 \times 10 = 180 \text{ N}$$

F. B. D. of B:



$$\text{Acceleration of B} = \frac{120 - 100}{40} = 0.5 \text{ m/s}^2 \text{ (backward)}$$

3. (c) The free body diagram of block in fig. b is as shown.



$$\therefore N = mg + P \cos \theta$$

and acceleration of block

$$a = \frac{F + P \sin \theta - \mu (mg + P \cos \theta)}{m}$$

$$a = \frac{F + P \sin \theta - \frac{\sin \theta}{\cos \theta} (mg + P \cos \theta)}{m} = \frac{F - \mu mg}{m}$$

Hence P does not change acceleration.

4. (b) If we consider blocks 2 and 1 independently, then there accelerations would be for block (1)

$$a_1 = g \sin \theta - \mu_1 g \cos \theta = g \left[\frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} \right] = \frac{g[2\sqrt{3} - 1]}{4}$$

For block (2)

$$a_2 = g \sin \theta - \mu_2 g \cos \theta = g \left[\frac{\sqrt{3}}{2} - \frac{2}{5} \times \frac{1}{2} \right] = \frac{g}{10} [5\sqrt{3} - 2]$$

Since $a_2 > a_1$ so both blocks will move separately.

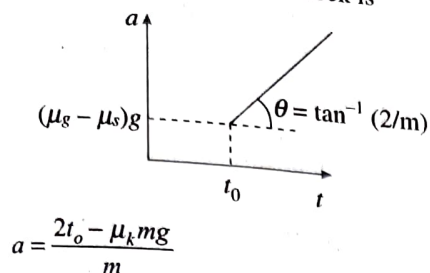
5. (d) Let t_0 be the time when friction force is maximum

$$F = 2t_0 = \mu_s mg$$

The block just starts moving immediately after this instant, with acceleration

$$= \frac{\mu_s mg - \mu_k mg}{m} = (\mu_s - \mu_k)g$$

For $t > t_0$ the acceleration of the block is



Multiple Correct Answers Type

6. (b, d)

Case I:

Since, no relative motion:

$$a = \frac{F_1 - F_f}{5} = \frac{F_f}{3} \Rightarrow F_{1(\max)} = \frac{8}{3}F_f$$

Case II:

$$a = \frac{F_f}{5} = \frac{F_2 - F_f}{3} \Rightarrow F_{2(\max)} = \frac{8}{5}F_f$$

Clearly, $F_{1(\max)} > F_{2(\max)}$ and $\frac{F_{1(\max)}}{F_{2(\max)}} = \frac{5}{3}$

Comprehension Type

Sol. 7. (a) 8. (d) 9. (b)

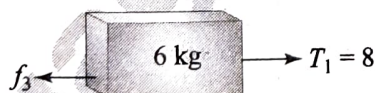


$$T_2 + 4 = 20, T_2 = 16 \text{ N}$$



$$f_2 = 8, T_2 = T_1 + f_2, T_2 = T_1 + 8$$

$$T_1 = 8$$



$$f_2 = 8$$

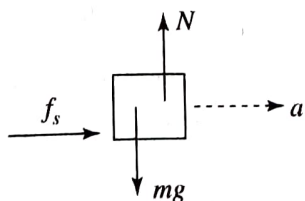
10. (a) FBD: The forces acting on m are $mg \downarrow, f_s \rightarrow, N \uparrow$

Force equation:

$$\Sigma F_x = f_s = ma_x = ma \quad (i)$$

$$\Sigma F_y = N = mg = ma_y = 0$$

Law of static friction, $f_s \leq \mu_s N$



(ii)

(iii)

Substituting f_s from equation (ii) and N from equation (i) in equation (iii), $ma \leq \mu_s mg$

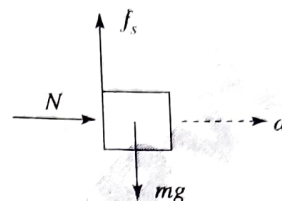
$$\Rightarrow a \leq \mu_s g$$

11. (b) FBD: The forces acting on m are $mg \downarrow, f_s \uparrow, N \rightarrow$

Force equation:

$$\Sigma F_x = N = ma_x = ma \quad (i)$$

$$\Sigma F_y = f_s - mg = ma_y = 0 \quad (ii)$$



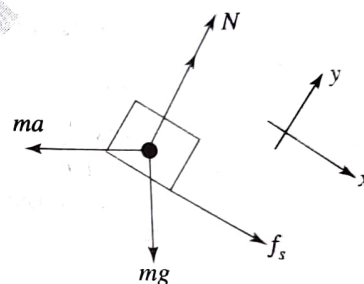
Law of $f_s: f_s \leq \mu_s N$

(iii)

Substituting f_s from equation (ii) and N from equation (i) in equation (iii),

$$mg \leq \mu_s ma \Rightarrow a \geq \frac{g}{\mu_s}$$

12. (d) Since the block is at rest relative to the accelerating frame.



Force equation:

$$\Sigma F_x = f_s + mg \sin \theta - ma \cos \theta = 0 \quad (i)$$

$$\Sigma F_y = N - ma \sin \theta - mg \cos \theta = 0 \quad (ii)$$

Law of $f_s: f_s \leq \mu_s N$

(iii)

Substituting f_s from equation (i) and N from equation (ii) in equation (iii),

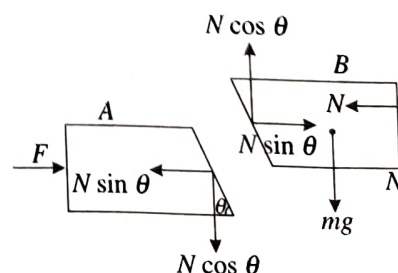
$$m(a \cos \theta - g \sin \theta) \leq \mu m (a \sin \theta + g \cos \theta)$$

$$a \leq g \left(\frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \right)$$

DPP 7.4

Subjective Type

1. (i) The FBDs of A and B are



For A to be in equilibrium;

$$F = N \sin \theta \quad (1)$$

For B to just lift off;

$$N \sin \theta = mg + \mu_s N' \quad (2)$$

For horizontal equilibrium of B;

$$N' = N \sin \theta \quad (3)$$

From (2) and (3)

$$N(\cos \theta - \mu_s \sin \theta) = mg \quad \text{or} \quad N = \left(\frac{4}{5} - \frac{2}{3} \times \frac{3}{5} \right) mg$$

$$\text{or } N = \frac{5}{2} mg \quad (4)$$

$$\text{From equation (1) } F = N \times \frac{3}{5} \quad \therefore F = \frac{3}{2} mg$$

(ii) The acceleration of the block A be a and B be b

$$F - N \sin \theta = 2ma \quad (1)$$

$$N \cos \theta - mg - \mu_s N' = mb \quad (2)$$

$$N' = N \sin \theta \quad (3)$$

From constraint =

$$a \sin \theta = b \cos \theta \quad (4)$$

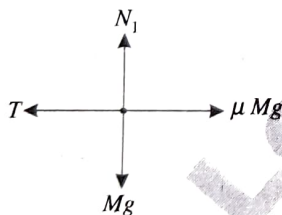
Solving (1), (2), (3) and (4) we get

$$\Rightarrow b = \frac{3g}{7}$$

2. (a) Let the acceleration of blocks of mass M and $3M$ are a_1 and a_2 respectively. From constraint relation

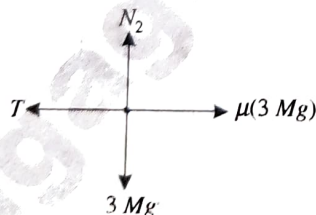
$$a_{\text{pulley}} = \frac{a_1 + a_2}{2} = a \quad (i)$$

F.B.D. of block of mass M



$$T - \mu Mg = Ma_1 \quad (ii)$$

F.B.D. of block of mass $3M$



$$T - 3\mu Mg = 3Ma_2 \quad (iii)$$

From (ii) and (iii):

$$-\mu Mg + 3\mu Mg = Ma_1 - 3Ma_2$$

$$\Rightarrow 2\mu g = a_1 - 3a_2 \quad (iv)$$

From (i) and (iv)

$$2\mu g = (2a - a_2) - 3a_2$$

$$\Rightarrow a_2 = \frac{a - \mu g}{2} \quad (v)$$

From (i) and (v)

$$a_1 = 2a - a_2 = 2a - \left(\frac{a - \mu g}{2} \right) = \frac{3a}{2} + \frac{\mu g}{2} = \frac{3a + \mu g}{2}$$

(b) Since $a = \frac{a_1 + a_2}{2}$

Acceleration a will be maximum when a_1 is maximum (since $a_2 = 0$)

Acceleration a_1 is maximum when tension in the string is maximum

For block of mass $3M$ to be stationary

$$T \leq 3\mu Mg$$

$$T_{\text{max}} = 3\mu Mg$$

For block of mass M

$$T_{\text{max}} - \mu Mg = Ma_1$$

$$\Rightarrow 3\mu Mg - \mu Mg = Ma_1$$

$$\Rightarrow 2\mu g = a_1$$

So maximum acceleration

$$a = \frac{a_1 + a_2}{2} = \frac{2\mu g + 0}{2} = \mu g$$

Single Correct Answer Type

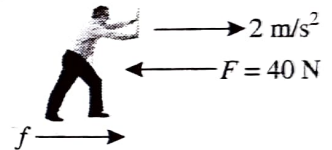
3. (b) For Block $F = 20.2 = 40$ N. As

boy exerts 40 N force on block, block exerts 40 N force on the boy, in opposite direction. As boy is also moving with same acceleration $-40 = 50.2$

$$f = 140 \text{ N}$$

Aliter: Consider the boy + block system. The only external force is friction acting on boy ' f '

$$\therefore f = (M_{\text{boy}} + M_{\text{block}})a = 140 \text{ N}$$



4. (c) Let m_A and m_B be the mass of blocks A and B respectively.

As the force F increases from 0 to $\mu_s m_A g$, the frictional force f on block A is such that $f = F$. When $F = \mu_s m_A g$, the frictional force f attains maximum value $f = \mu_s m_A g$.

As F is further increased to $\mu_s (m_A + m_B)g$, the block A does not move. In this duration frictional force on block A remains constant at $\mu_s m_A g$.

Hence (c) is correct choice.

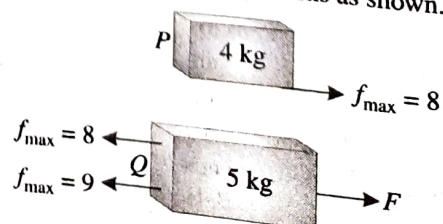
$$5. (d) F - 8(0.42)(10) - 2(0.42)(10) = 6(1.5)$$

$$F - 42 = 9$$

$$F = 51 \text{ N}$$

6. (c) So block 'Q' is moving due to force while block 'P' due to friction.

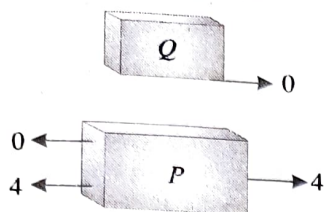
Friction direction on both +Q blocks as shown.



First block 'Q' will move and P will move with 'Q' so by FBD taking 'P' and 'Q' as system

$$F - 9 = 0 \Rightarrow F = 9 \text{ N}$$

When applied force is 4 N then FBD



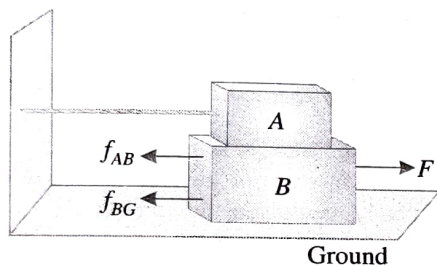
4 kg block is moving due to friction and maximum friction force is 8 N.

$$\text{So acceleration} = \frac{8}{4} = 2 \text{ m/s}^2 = a_{\max}$$

Slipping will start at when Q has +ve acceleration equal to maximum acceleration of P, i.e., 2 m/s^2 .

$$F - 17 = 5 \times 2 \Rightarrow F = 27 \text{ N.}$$

7. (c)



$$\begin{aligned} F &= f_{AB} + f_{BG} \\ &= \mu_{AB} m_A g + \mu_{BG} (m_A + m_B) g \\ &= 0.2 \times 100 \times 10 + 0.3(300) \times 10 \\ &= 200 + 900 = 1100 \text{ N} \end{aligned}$$

8. (a) There is no friction between the body B and surface of the table. If the body B is pulled with force F, then

$$F = (m_A + m_B)a$$

Due to this force upper body A will feel the pseudo force in a backward direction.

$$f = m_A \times a$$

But due to friction between A and B, body will not move. The body A will start moving when pseudo force is more than friction force.

$$\text{i.e. for slipping, } m_A a = \mu m_A g \therefore a = \mu g$$

9. (a) Limiting friction between block and slab = $\mu_s m_A g$

$$= 0.6 \times 10 \times 10 = 60 \text{ N}$$

But applied force on block A is 100 N. So the block will slip over a slab.

Now kinetic friction works between block and slab $f_k = \mu_k m_A g$

$$= 0.4 \times 10 \times 10 = 40 \text{ N}$$

This kinetic friction helps to move the slab

$$\therefore \text{Acceleration of slab} = \frac{40}{m_B} = \frac{40}{40} = 1 \text{ m/s}^2$$

10. (d) Maximum frictional force between C and ground = 300 N
Max. frictional force between B and ground = 360 N
So man is unable to pull B. Hence $T = 0$.

11. (c) In equilibrium

$$T = mg$$

$$N = 3 mg$$

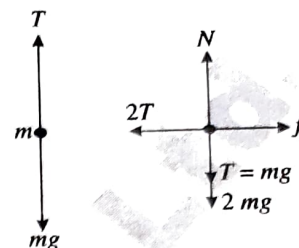
$$\text{and } f = 2T = 2 mg$$

In limiting case $f < f_{\max}$

$$\Rightarrow 2mg \leq \mu N$$

$$\Rightarrow 2mg \leq 3\mu mg$$

$$\Rightarrow \mu > \frac{2}{3}$$



Matching Column Type

12. The acceleration of two block system for all cases is $a = 2 \text{ m/s}^2$

In option (p) the net force on 2 kg block is frictional force

$\therefore$ Frictional force on 2 kg block is

$$f = 2 \times 2 = 4 \text{ N towards right}$$

In option (q) the net force on 4 kg block is frictional force

$\therefore$ Frictional force on 4 kg block is

$$f = 4 \times 2 = 8 \text{ N towards right}$$

In option (r) the net force on 2 kg block is $2 \times 2 = 4 \text{ N}$

$\therefore$ Friction force f on 2 kg block is towards left.

$$\therefore 6 - f = 2 \times 2 \text{ or } f = 2 \text{ N}$$

In option (s) the net force on 2 kg block is ma

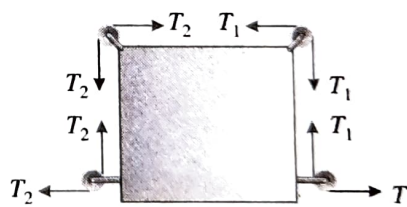
$$= 2 \times 2 = 4 \text{ N towards right.}$$

$\therefore$ Friction force on 2 kg block is 12 N towards right.

$$\therefore \text{required force } F = (m + m + m + M) a = 2 \mu mg$$

Comprehension Type

13. (d) The free body diagram of cart is



Hence net horizontal force on cart is zero.

$\therefore$ acceleration of cart is zero.

14. (c) The acceleration of each block is equal and equal to $\frac{F}{3m}$.

$\therefore$ Tension in required string can be found by applying Newton's second law to block C.

$$T_2 = ma = \frac{F}{3}$$

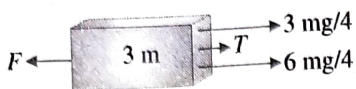
15. (b) For block B to just slip on the cart, the friction force on cart is μmg . The net force on cart is thus μmg . Hence acceleration

$$\text{of cart is } a = \frac{\mu mg}{3m} = \frac{\mu g}{3}$$

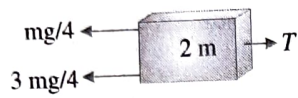
DPP 7.5

Subjective Type

1.



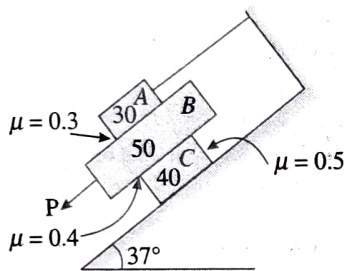
$$F - \frac{3mg}{4} - \frac{6mg}{4} - T = 0$$



$$T - \frac{mg}{4} - \frac{3mg}{4} = 0$$

$$(1) \text{ and } (2) \Rightarrow F = 13 \frac{mg}{4}$$

$$2. f_{AB \max} = 0.3 \times 30 \text{ g} \cos 37^\circ$$



$$f_{AB \max} = 72 \text{ N}$$

$$f_{BC \max} = 0.4 \times 80 \text{ g} \cos 37^\circ = 256 \text{ N}$$

$$f_{C \max} = 0.5 \times 120 \text{ g} \cos 37^\circ = 480 \text{ N}$$

when 'B' block is pulled two cases are possible

(1) A & C remains stationary only 'B' tends to move downwards.

(2) B & C both moves

together and there

is just slipping and

A & B contact and C

and wedge contact

Taking case (2)

Let B & C moves

together and there

is just slipping

between A & B and

between wedge and C

$$\Rightarrow P + 40 \text{ g} \sin 37^\circ + 50 \text{ g} \sin 37^\circ - 72 - 480 = 0$$

$$\Rightarrow P = 12 \text{ N}$$

checking friction between B & C

$$f_{BC} + 40 \text{ g} \sin 37^\circ - 480 = 0$$

$$f_{BC} = 240$$

which is within limit so case

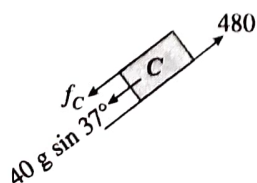
is correct so max. force

for which there will be no

slipping $P_{\max} = 12 \text{ N}$ the at

this force both B & C tends

to move together.



Single Correct Answer Type

3. (c) FBD of B



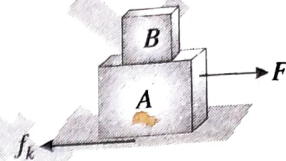
$$(a_B)_{\max} = \frac{f_{\max}}{m_B} = \mu_s g = 2.5 \text{ m/s}^2$$

FBD of combined system

$$f_k = 0.15(2 + 10)g = 18 \text{ N}$$

$$F_{\max} - f_k = (m_A + m_B)(a_B)_{\max}$$

$$\Rightarrow F_{\max} = f_k + 12 \times 2.5 = 48 \text{ N}$$



4. (a) The sliding shall start at lower surface first

$$\text{if } F > 0.5 [10 + 10] \text{ g}$$

$$\text{or } F > 100 \text{ N}$$

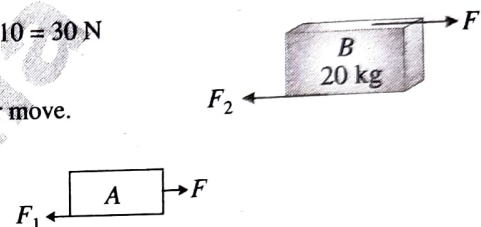
$$5. (d) F_{1(\max)} = 0.2 \times 10 \times 10 = 20 \text{ N}$$

$$F_2 = 0.1 \times 30 \times 10 = 30 \text{ N}$$

$$F_{1(\max)} < F_{2(\max)}$$

$\Rightarrow$ 'B' can never move.

$$6. (b) F_{1(\max)} = 20 \text{ N}$$



$\therefore F \geq 20 \text{ N}$, to make 'A' slip on 'B'.

7. (b) Let f_1 be the magnitude of limiting frictional force between the blocks and f_2 be the magnitude of limiting frictional force between the block and horizontal surface. The free body diagrams for both the blocks is as shown. Applying Newton's second law to both the blocks

$$F - f_1 - T = m_A a \quad (1)$$

$$F - f_1 - f_2 = m_B a \quad (2)$$

For F to just pull the blocks, acceleration of blocks $a = 0$.

Therefore from equation (1) and (2),

$$F = 2f_1 + f_2 \text{ or } 25 = \mu(2 \times 1 \times 10 + 2 \times 10)$$

solving we get $\mu = 0.5$

8. (d) (i) If both moves together $a = 2 \text{ m/s}^2$

Force required for A = 4 N

Max. friction force = 10 N

Hence there will be no slipping and friction force will be 4 N.

(ii) Max. Friction force = $\mu mg \cos \theta = 8 \text{ N}$

Force along incline = $mg \sin \theta = 12 \text{ N}$

Hence block will move and friction force will be 8 N.

(iii) Max. friction force = $\mu N = 5 \text{ N}$

Downward force = 20 N

Block will slip and friction force will be 5 N

(iv) Acceleration of the system = $\frac{10}{6} \text{ m/s}^2$

$$\text{Force required for A} = \frac{20}{6} \text{ N}$$

Max. friction force = 10 N

Hence A and B will move together and friction force will be $\frac{20}{6} \text{ N}$.

9. (c) Wedge will start slipping when

$$F \geq \mu(2 \text{ mg})$$

10. (b) If wedge start slipping, common acceleration

$$a_c = \frac{F - 2\mu mg}{2m}$$

If $mg \sin \theta = ma \cos \theta$, then no force along the plane will be felt by the block and hence friction will be zero.

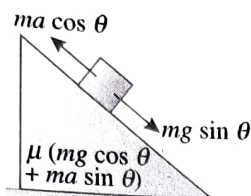
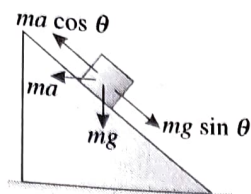
$$\Rightarrow mg \sin \theta = m \left(\frac{F - 2\mu mg}{2m} \right) \cos \theta$$

$$F = 2mg \tan \theta + 2\mu mg$$

11. (b) Block will start sliding if

$$ma \cos \theta \geq mg \sin \theta + \mu(mg \cos \theta + ma \sin \theta)$$

$$\text{get } a > g \left(\frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \right)$$



Multiple Correct Answers Type

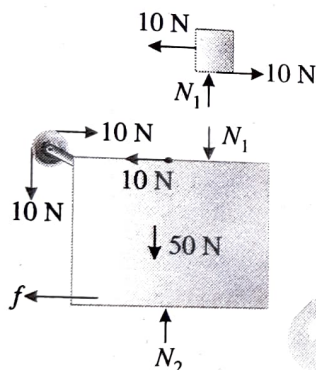
12. (a, d)

The frictional force on block A is

$$\Rightarrow \mu N_1 = 10 \Rightarrow N_1 = \frac{10}{0.2} = 50 \text{ N}$$

The net force on block B in vertical direction is zero

$$\therefore N_1 = 50 + N_1 + 10 = 110 \text{ N}$$



$\Rightarrow$ Normal reaction exerted by ground on block B is 110 N.

The net force on block B in horizontal direction is zero.

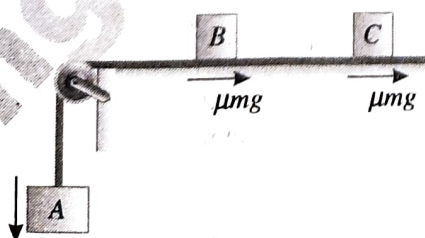
$$\therefore f + 10 - 10 = 0$$

$\Rightarrow$ frictional force exerted by ground on block B is zero

13. (a, c)

Maximum possible acceleration of block will be $= \mu g$.

- (i) When $\mu = 0.2$ maximum acceleration possible so that block does not slip is 2 m/s^2 . Now acceleration of system assuming blocks don't slip is



$$a = \frac{40g - 2\mu mg}{100} = 3.8 \text{ m/s}^2$$

$$\mu = 0.2$$

As $3.8 > 2$ hence the assumption is wrong and blocks slip on the belt.

$\therefore$ Maximum force applicable on block B is μmg

$\therefore$ Acceleration of block $= 2 \text{ m/s}^2$

Hence velocity of B when it strikes pulley is $V^2 = 0^2 + 2 \times 2 \times 2$

$$V^2 = 8$$

$$\therefore V = 2\sqrt{2} \text{ m/s}$$

Hence (a).

- (ii) When $\mu = 0.5$ maximum acceleration possible so that block does not slip is 5 m/s^2

Now acceleration of system assuming blocks don't slip is

$$a = \frac{40g - 2\mu mg}{100} = 1 \text{ m/s}^2$$

$$\mu = 0.5$$

as $1 < 5$ hence, blocks do not slip and acceleration of block B is 1 m/s^2

$\therefore$ Hence velocity of B when it strikes pulley is

$$V^2 = 0^2 + 2 \cdot 1 \cdot 2$$

$$V^2 = 4$$

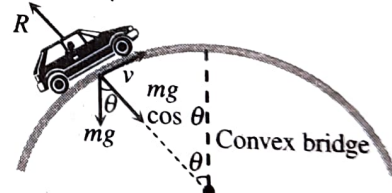
$$\therefore V = 2 \text{ m/s}$$

Hence (c).

DPP 7.6

Single Correct Answer Type

$$1. (a) R = mg \cos \theta - \frac{mv^2}{r}$$



when θ decreases $\cos \theta$ increases i.e., R increases.

2. (c) T = tension, W = weight and F = centrifugal force.

3. (d) The particle is moving in circular path

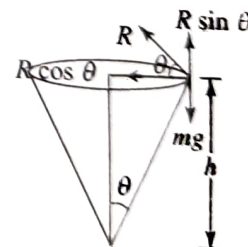
From the figure, $mg = R \sin \theta$ (i)

$$\frac{mv^2}{r} = R \cos \theta \quad (ii)$$

From equation (i) and (ii) we get

$$\tan \theta = \frac{rg}{mv^2} \text{ but } \tan \theta = \frac{r}{h}$$

$$\therefore h = \frac{v^2}{g} = \frac{(0.5)^2}{10} = 0.025 \text{ m} = 2.5 \text{ cm}$$



$$4. (c) \text{ Tension, } T = \frac{mv^2}{r} + mg \cos \theta$$

$$\text{For, } \theta = 30^\circ, T_1 = \frac{mv^2}{r} + mg \cos 30^\circ$$

$$\theta = 60^\circ, T_2 = \frac{mv^2}{r} + mg \cos 60^\circ \therefore T_1 > T_2$$

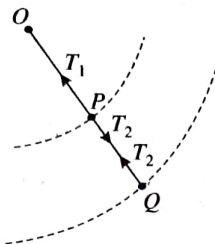
5. (b) We know that centripetal force $= m\omega^2 r$. Tension in the string between P and O is equal to the tension in the string between the spheres P and Q + centripetal force required for the sphere P .

In other words, we can form the equations of force.

For P , $T_1 - T_2 = m \times 1 \omega^2$

For Q , $T_2 = m \times 2 \omega^2$

Adding, $T_1 = 3m\omega^2 \Rightarrow \frac{T_2}{T_1} = \frac{2m\omega^2}{3m\omega^2} = \frac{2}{3}$



6. (a) In this problem it is assumed that particle although moving in a vertical loop, its speed remain constant.

Tension at lowest point $T_{\max} = \frac{mv^2}{r} + mg$

Tension at highest point $T_{\min} = \frac{mv^2}{r} - mg$

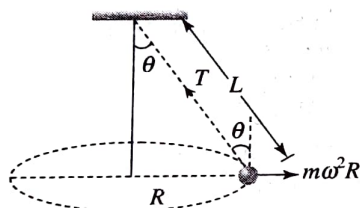
$$\frac{T_{\max}}{T_{\min}} = \frac{\frac{mv^2}{r} + mg}{\frac{mv^2}{r} - mg} = \frac{5}{3}$$

by solving we get,

$$v = \sqrt{4gr} = \sqrt{4 \times 10 \times 2.5} = 10 \text{ m/s}$$

7. (d) $T \sin \theta = M\omega^2 R$

$$T \sin \theta = M\omega^2 L \sin \theta$$

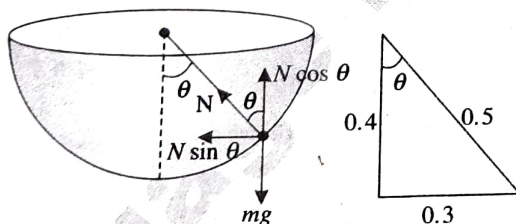


From (i) and (ii)

$$T M \omega^2 L = M 4 \pi^2 n^2 L = M 4 \pi^2 \left(\frac{2}{\pi}\right)^2 L = 16 ML$$

8. (c) $N \sin \theta = m\omega^2 r$

$$N \sin \theta = m\omega^2 r$$



(a)

(b)

$$\tan \theta = \frac{r\omega^2}{g}$$

$$\omega = \sqrt{\frac{g \tan \theta}{r}}$$

$$\omega = 5 \text{ rad/s}$$

Multiple Correct Answers Type

9. (b, c, d)

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(v^2/R)^2 + \beta^2}$$

Net acceleration is not towards centre.

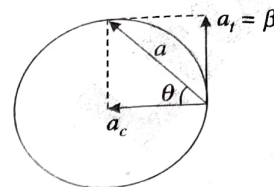
$$f = ma > ma_c \Rightarrow f > \frac{mv^2}{R}$$

$$\Rightarrow f_c = \mu mg \geq f > ma_c$$

$$\Rightarrow \mu mg > m\beta \Rightarrow \mu > \frac{\beta}{g}$$

$$\text{also } \mu mg > f > ma_c \Rightarrow \mu mg > \frac{mv^2}{R}$$

$$\Rightarrow \mu > \frac{v^2}{Rg}$$



10. (a, c)

Centripetal force $= \frac{mv^2}{r}$ and is directed always towards the centre of circle. Sense of rotation does not affect magnitude and direction of this centripetal force.

Matching Column Type

11. (p $\rightarrow$ d) (q $\rightarrow$ a, c, d) (r $\rightarrow$ b) (s $\rightarrow$ c)

- (p) as speed is constant, so net force should be always $\frac{mv^2}{r}$ (towards center) which is constant in magnitude.

$$|\vec{N} + \vec{w} + \vec{f}| = \frac{mv^2}{r} \rightarrow \text{constant}$$

- (q) $\vec{N} + \vec{w} + \vec{f}$ will be directed towards center.

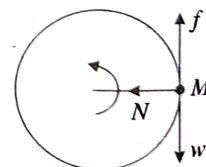
$\vec{N}$ will always be directed towards center.

So $\vec{w} + \vec{f}$ will always be directed towards or away from center depending upon the location of motorcycle.

- (r) Total reaction force by track is the resultant of $\vec{N}$ and $\vec{f}$ which is $\vec{N} + \vec{f}$.

- (s) At M , motion is along vertical. At M net tangential force should be zero.

$$\vec{f} + \vec{w} = 0$$



Comprehension Type

Sol.

12. (a) 13. (b)

$$T \cos \theta = mg$$

$$\text{and } T \sin \theta = m\omega^2 (R + L \sin \theta)$$

From equation (i)

$$T = mg \sec \theta$$

Solving equation (i) and (ii)

$$\omega = \sqrt{\frac{g \tan \theta}{R + L \sin \theta}}$$

(iii)

DPP 8.1

Single Correct Answer Type

1. (b) Work done does not depend on time.
2. (b) Work done by centripetal force is always zero because force and instantaneous displacement are always perpendicular.

$$W = \vec{F} \cdot \vec{s} = Fs \cos \theta = Fs \cos(90^\circ) = 0$$

3. (d) $s = \frac{t^2}{4} \therefore ds = \frac{t}{2} dt$

$$F = ma = \frac{md^2s}{dt^2} = \frac{6d^2}{dt^2} \left[\frac{t^2}{4} \right] = 3N$$

Now

$$W = \int_0^2 F ds = \int_0^2 3 \frac{t}{2} dt = \frac{3}{2} \left[\frac{t^2}{2} \right]_0^2 = \frac{3}{4} [(2)^2 - (0)^2] = 3J$$

4. (b) $W = \int_0^{x_1} F \cdot dx = \int_0^{x_1} Cx dx = C \left[\frac{x^2}{2} \right]_0^{x_1} = \frac{1}{2} Cx_1^2$

5. (c) When the block moves vertically downward with acceleration $\frac{g}{4}$, then tension in the cord

$$T = M \left(g - \frac{g}{4} \right) = \frac{3}{4} Mg$$

Work done by the cord

$$= \vec{F} \cdot \vec{s} = Fs \cos \theta$$

$$= Td \cos(180^\circ) = - \left(\frac{3Mg}{4} \right) \times d = -3Mg \frac{d}{4}$$

6. (c) $W = \frac{F^2}{2k}$

If both springs are stretched by same force then $W \propto \frac{1}{k}$

As $k_1 > k_2$ therefore $W_1 < W_2$

i.e. more work is done in case of second spring.

7. (d) $S = \frac{t^3}{3} \therefore dS = t^2 dt$

$$a = \frac{d^2 S}{dt^2} = \frac{d^2}{dt^2} \left[\frac{t^3}{3} \right] = 2t \text{ m/s}^2$$

Now work done by the force $W = \int_0^2 F \cdot dS = \int_0^2 ma \cdot dS$

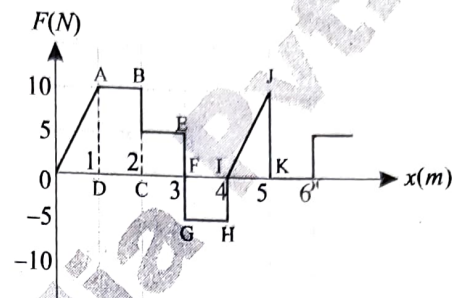
$$\int_0^2 3 \times 2t \times t^2 dt = \int_0^2 6t^3 dt = \frac{3}{2} [t^4]_0^2 = 24J$$

8. (a) From the graph it is clear that force is acting on the particle in the region AB and due to this force kinetic energy (velocity) of the particle increases. So the work done by the force is positive.

9. (b) Work done

= area under F - x graph

= area of rectangle ABCD + area of rectangle LCEF
+ area of rectangle GFHI + area of triangle IJK



$$= (2-1) \times (10-0) + (3-2)(5-0) + (4-3)(-5-0) + \frac{1}{2} (5-4)(10-0) = 15J$$

10. (d) $\int \vec{F} \cdot d\vec{x} = \int A(y^2 \hat{i} + 2x^2 \hat{j}) \cdot (dx \hat{i} + dy \hat{j})$

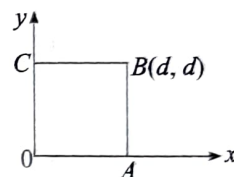
$$= A \int (y^2 dx + 2x^2 dy)$$

$$W_{OA} = 0 + 0, W_{AB} = A[0 + 2d^2 d]$$

$$W_{BC} = A[d^2(-d) + 0], W_{CD}$$

$$= A[0 + 0]$$

$$W = 0 + 2Ad^3 - Ad^3 + 0 = Ad^3$$



11. (c) While moving from (0, 0) to (a, 0)

Along positive x -axis, $y = 0 \therefore \vec{F} = -kx\hat{j}$

i.e., force is in negative y -direction while displacement is in positive x -direction.

$$\therefore W_1 = 0$$

Because force is perpendicular to displacement

Then particle moves from (a, 0) to (a, a) along a line parallel to y -axis ($x = +a$) during this $\vec{F} = -k(y\hat{i} + a\hat{j})$

The first component of force, $-ky\hat{i}$ will not contribute any work because this component is along negative x -direction ($-\hat{i}$) while displacement is in positive y -direction (a, 0) to (a, a). The second component of force i.e. $-ka\hat{j}$ will perform negative work

$$\therefore W_2 = (-ka\hat{j}) \cdot (a\hat{j}) = (-ka)(a) = -ka^2$$

So net work done on the particle $W = W_1 + W_2$

$$= 0 + (-ka^2) = -ka^2$$

Comprehension Type

12. (d) Displacement of block in the time interval T is $\frac{1}{2}bT^2$

$$W_{\text{gravity}} = (-mg) \left(+\frac{1}{2}bT^2 \right) = -\frac{1}{2}mgbT^2$$

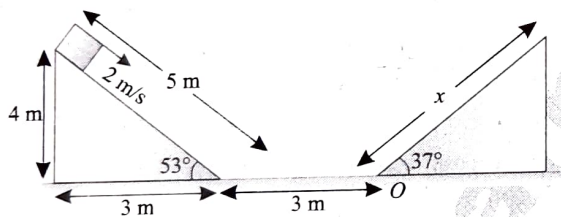
13. (c) The normal reaction acting on the block is $M(g + b)$ in upward direction. The upward displacement of the block is $\frac{1}{2}bT^2$.
Hence the work done by normal reaction is $M(g + b) \times \frac{1}{2}bT^2$.

14. (d) According to observer B, the displacement of the block is zero, therefore
Work done by gravity is zero
Work done by normal reaction is zero
Work done by pseudo force is zero.

DPP 8.2

Subjective Type

1. Let x be the distance travelled by the particle along the 37° incline, after which it comes to rest. From work-energy theorem;
work done by gravity on block on incline 53°
+ work done by friction on block on incline 53°
+ work done by friction on ground level + work done by gravity on block on incline 37° + work done by friction on block on incline 37° = change in K.E.
 $\Rightarrow mg(4) - \mu mg \cos 53^\circ(5) - \mu mg(3) - \mu mg \cos 37^\circ(x) - mg \sin 37^\circ(x) = 0 - \frac{1}{2}(1)(2)^2$



$$\Rightarrow 40 - 0.5 \times 10 \times \frac{3}{5} \times 5 - 0.5 \times 10 \times 3 - 0.5 \times 10 \times \frac{4}{5}(x) - \frac{3}{5}(x) = 0 - 2$$

On solving we get $x = 1.2$ m

2. Force on cone while it is penetrating the sand is shown in F.B.D. below
Applying work energy theorem to the cone as x changes from 0 to d
 $\Delta KE = \text{work done by } mg + \text{work done by resistive force } R$



$$K_{\text{final}} - K_{\text{initial}} = mgd - \int_0^d kx^2 dx$$

$$0 - mgh = mgd - \int_0^d kx^2 dx$$

$$\therefore \frac{kd^3}{3} = (mgd + gh) \Rightarrow k = \frac{3mg}{d^3}(h + d)$$

Single Correct Answer Type

3. (a) As ΔKE is same in both the cases, work done will be same.

4. (b) From work energy theorem

$$Fd = K_f - K_i = K_f$$

$$\text{so } K_1 = K_2$$

$$\therefore m_1 > m_2$$

$$\text{so } p_1 > p_2$$

5. (d) Initial KE of the body = $\frac{1}{2}mv^2 = \frac{1}{2} \times 25 \times 4 = 50$ J

Work done against resistive force

$$= \text{Area between } F-x \text{ graph}$$

$$= \frac{1}{2} \times 4 \times 20 = 40$$
 J

$$\text{Final KE} = \text{Initial KE} - \text{Work done against resistive force} \\ = 50 - 40 = 10$$
 J

6. (d) Work done = change in kinetic energy

$$W = \frac{1}{2}mv^2 \therefore W \propto v^2 \text{ graph will be parabolic in nature}$$

7. (c) When body moves under action of constant force then kinetic energy acquired by the body $K.E. = F \times S$

$$\therefore KE \propto S \text{ (If } F = \text{constant)}$$

So the graph will be straight line.

8. (d) Increase in KE = work done

$$\frac{1}{2}mv_2^2 - \frac{1}{2}m \times \left(\frac{2F_0x_0}{m} \right) = \frac{1}{2} \times 2F_0 \cdot 3x_0 + 3F_0x_0$$

$$\Rightarrow v_2 = \sqrt{\frac{14F_0x_0}{m}}$$

Multiple Correct Answers Type

9. (a, b, d)

Free body diagram of block is as shown in figure.

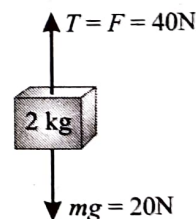
From work-energy theorem

$$W_{\text{net}} = \Delta KE \text{ or } (40 - 20)s = 40$$

$$s = 2$$
 m

Work done by gravity is $-20 \times 2 = -40$ J

and work done by tension is $40 \times 2 = 80$ J



10. (a, b, c)

Maximum extension will be at the moment when both masses stop momentarily after going down. Applying W-E theorem from starting to that instant.

$$k_f - k_i = W_{\text{gr.}} + W_{\text{sp}} + W_{\text{ten.}}$$

$$0 - 0 = 2Mgx + \left(-\frac{1}{2}Kx^2 \right) + 0$$

$$x = \frac{4Mg}{K}$$

System will have maximum KE when net force on the system becomes zero. Therefore

$$2Mg = T \text{ and } T = kx$$

$$\Rightarrow x = \frac{2Mg}{K}$$

Hence KE will be maximum when $2M$ mass has gone down by $\frac{2Mg}{K}$.

Applying W/E theorem

$$k_f - 0 = 2Mg \frac{2Mg}{K} - \frac{1}{2}K \cdot \frac{4M^2g^2}{K^2}$$

$$k_f = \frac{2M^2g^2}{K^2}$$

Maximum energy of spring

$$\frac{1}{2}K \cdot \left(\frac{4Mg}{K}\right)^2 = \frac{8M^2g^2}{K}$$

Therefore maximum spring energy = $4 \times$ maximum KE

When KE is maximum $x = \frac{2Mg}{K}$.

$$\text{Spring energy} = \frac{1}{2} \cdot K \cdot \frac{4M^2g^2}{K^2} = \frac{2M^2g^2}{K^2}$$

i.e. (D) is wrong.

Comprehension Type

$$11. (a) AC = \sqrt{(2R)^2 + \left(\frac{3}{2}R\right)^2} = \frac{5}{2}R$$

$$\text{Extension in length of spring } (x) = \frac{5}{2}R = 2R = \frac{R}{2}$$

$$\text{Work done by spring} = \frac{1}{2}kx^2$$

$$= \frac{1}{2} \left(\frac{4mg}{R} \right) \left(\frac{R^2}{4} \right) = \frac{mgR}{2}$$

12. (a) Let v be velocity of the ring as it reaches the ground.
From work energy theorem;

$$W_{\text{gravity}} + W_{\text{spring}} = \Delta K$$

$$\Rightarrow mg \left(\frac{3R}{2} \right) + \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

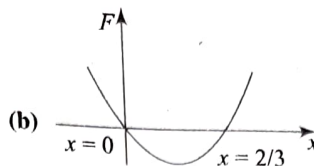
$$\Rightarrow \frac{3mgR}{2} + \frac{mgR}{2} = \frac{1}{2}mv^2$$

$$\Rightarrow v = 2\sqrt{gR}$$

DPP 8.3

Subjective Type

1. (a) The particle is at equilibrium at $x = 0$ and $x = \frac{2}{3}$ m.



The particle is in stable equilibrium at $x = 0$ metre and unstable equilibrium at $x = \frac{2}{3}$ metre

(c) The minimum speed imparted to the particle should be such that it just reaches $x = \frac{2}{3}$ from there on it shall automatically reach $x = 0$

$$\frac{1}{2}mv^2 = - \int_0^{2/3} F dx = - \int_0^{2/3} x(3x-2) dx = \frac{1300}{27}$$

$$\text{or } v = \sqrt{\frac{2600}{27}} \text{ m/s}$$

Single Correct Answer Type

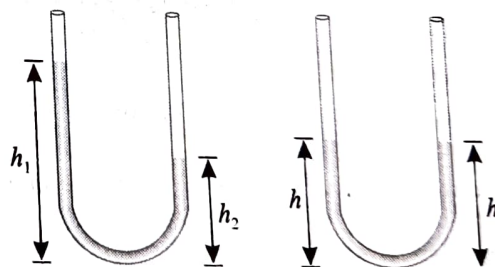
$$2. (b) U = A - Bx^2 \Rightarrow F = -\frac{dU}{dx} = 2Bx \Rightarrow F \propto x$$

$$3. (d) \text{ Condition for stable equilibrium } F = -\frac{dU}{dx} = 0$$

$$\Rightarrow -\frac{d}{dx} \left[\frac{a}{x^{12}} - \frac{b}{x^6} \right] = 0 \Rightarrow -12ax^{-13} + 6bx^{-7} = 0$$

$$\Rightarrow \frac{12a}{x^{13}} = \frac{6b}{x^7} \Rightarrow \frac{2a}{b} = x^6 \Rightarrow x = \sqrt[6]{\frac{2a}{b}}$$

4. (d)



If h is the common height when they are connected, by conservation of mass

$$\rho A_1 h_1 + \rho A_2 h_2 = \rho h(A_1 + A_2)$$

$$h = (h_1 + h_2)/2 \quad [\text{as } A_1 = A_2 = A \text{ given}]$$

As $(h_1/2)$ and $(h_2/2)$ are heights of initial centre of gravity of liquid in two vessels., the initial potential energy of the system

$$U_i = (h_1 A \rho) g \frac{h_1}{2} + (h_2 A \rho) g \frac{h_2}{2} = \rho g A \frac{(h_1^2 + h_2^2)}{2} \quad (i)$$

When vessels are connected the height of centre of gravity of liquid in each vessel will be $h/2$,

$$\text{i.e. } \frac{(h_1 + h_2)}{4} \quad [\text{as } h = (h_1 + h_2)/2]$$

Final potential energy of the system

$$U_F = \left[\frac{(h_1 + h_2)}{2} A \rho \right] g \left(\frac{h_1 + h_2}{4} \right) \\ = A \rho g \left[\frac{(h_1 + h_2)^2}{4} \right] \quad (ii)$$

Work done by gravity

$$W = U_i - U_f = \frac{1}{4} \rho g A [2(h_1^2 + h_2^2) - (h_1 + h_2)^2] \\ = \frac{1}{4} \rho g A (h_1 - h_2)^2$$

5. (b) Here $\frac{mv^2}{r} = \frac{K}{r^2} \therefore KE = \frac{1}{2} mv^2 = \frac{K}{2r}$

$$U = - \int_{\infty}^r F \cdot dr = - \int_{\infty}^r \left(-\frac{K}{r^2} \right) dr = -\frac{K}{r}$$

$$\text{Total energy } E = KE + PE = \frac{K}{2r} - \frac{K}{r} = -\frac{K}{2r}$$

6. (c) Work done = Gain in potential energy
Area under curve = mgh

$$\Rightarrow \frac{1}{2} \times 11 \times 100 = 5 \times 10 \times h$$

$$\Rightarrow h = 11 \text{ m}$$

7. (c) As slope of problem graph is positive and constant up to certain distance and then it becomes zero.

So from $F = \frac{-dU}{dx}$, up to distance a , $F = \text{constant (negative)}$
and becomes zero suddenly.

8. (b) When particle moves away from the origin then at position $x = x_1$ force is zero and at $x > x_1$, force is positive (repulsive in nature) so particle moves further and does not return back to original position.

i.e. the equilibrium is not stable.

Similarly at position $x = x_2$ force is zero and at $x > x_2$, force is negative (attractive in nature)

So particle returns back to original position i.e. the equilibrium is stable.

9. (c) $F = \frac{-dU}{dx}$ it is clear that slope of $U - x$ curve is zero at point

B and C . $\therefore F = 0$ for point B and C

10. (a) When the distance between atoms is large then interatomic force is very weak. When they come closer, force of attraction increases and at a particular distance force becomes zero. When they are further brought closer force becomes repulsive in nature.

This can be explained by slope of $U - x$ curve shown in graph

(a).

11. (c) At equilibrium position $x = \frac{mg}{k}$

$$U_{\text{spring}} = \frac{1}{2} kx^2 = \frac{1}{2} k \left(\frac{mg}{k} \right)^2 \cdot x = \frac{mgx}{2} \\ = \frac{1}{2} (\text{loss in G.P.E.})$$

$G = 2S$, in which potential energy increases

Multiple Correct Answers Type

12. (b, d)

At point 'C', the potential energy is minimum, hence it is a point of stable equilibrium.

Also, from E to F , the slope is negative i.e., $\frac{dU}{dr} < 0$

Hence, the force of interaction between the particles is repulsive between points E and F .

13. (a, b, c)

If the particle is released at the origin, it will try to go in the direction of force. Here $\frac{du}{dx}$ is positive and hence force is negative, as a result it will move towards $-ve$ x -axis.

When the particle is released at $x = 2 + \Delta$; it will reach the point of least possible potential energy (-15 J) where it will have maximum kinetic energy.

$$\therefore \frac{1}{2} mv_{\text{max}}^2 = 25 \Rightarrow v_{\text{max}} = 5 \text{ m/s}$$

The particle will now perform oscillatory motion between -15 J $\leq U \leq 15$ J, because reaching $U = +15$ J, the kinetic energy and hence speed becomes zero.

$$-15 \text{ J} \leq U \leq 15 \text{ J}, U = +15 \text{ J},$$

$$\text{In (C); } E_i = U_i + k_i = 15 + 6 = 21 \text{ J}$$

$$\text{At } x = 10 \text{ m; } U_f = 20 \text{ J} \Rightarrow K_f = 1 \text{ J} \neq 0 \Rightarrow x = 10 \text{ m}$$

$$\Rightarrow \text{The particle cross } x = 10 \text{ m.}$$

DPP 8.4

Subjective Type

$$1. \frac{1}{2} k \left(\frac{3L_0}{4} \right)^2 = \frac{1}{2} mv^2 + \frac{1}{2} k (L_0 - x)^2$$

when $x < L_0$

$$\Rightarrow v = \sqrt{\frac{k}{m} \left[\left(\frac{3L_0}{4} \right)^2 - (L_0 - x)^2 \right]}$$

when $x \geq L_0$

velocity is maximum when $x = L_0$ and

$$v_{\text{max}} = \frac{3L_0}{4} \sqrt{\frac{k}{m}}$$

2. A particle cannot be found in the given region when $KE < 0$.

We know that

$$\text{Total energy } E = PE + KE$$

$$\Rightarrow E = U + K$$

For region A Given $U > E$ from Eq. (i)

$$K = E - U$$

$$\text{as } U > E \Rightarrow E - U < 0$$

Hence $K < 0$, this is not possible.

For region B Given, $U < E \Rightarrow E - U > 0$

This is possible because total energy can be greater than PE (U).

For region C Given, $K > E \Rightarrow K - E > 0$

From Eq. (i) $PE = U = E - K < 0$

Which is possible because PE can be negative.

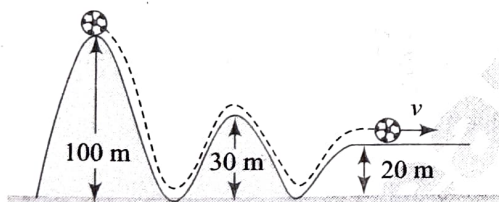
For region D Given, $U > K$

This is possible because for a system PE (U) may be greater than KE (K).

3. (a) As ball 1 is rolling down without slipping there is no dissipation of energy, hence total mechanical energy is conserved. Ball 3 is having negligible friction, hence there is no loss of energy.
- (b) Ball 1 acquires rotational energy, ball 2 loses energy by friction. They cannot cross at C. Ball 3 can cross over.
- (c) Balls 1 and 2 turn back before reaching C. Because of loss of energy, ball 2 cannot reach back to A. Ball 1 has a rotational motion in "wrong" sense when it reaches B. It cannot roll back to A because of kinetic friction.

Single Correct Answer Type

4. (d) Work done by kinetic friction on a body may increase its kinetic energy.
As for (b), it is true for conservative forces.
For (c), internal forces must be conservative and work of external forces on the system must be zero.
5. (a) The force is constant and hence conservative
 $\therefore W_1 = W_2$
6. (c)



Ball starts from the top of a hill which is 100 m high and finally rolls down to a horizontal base which is 20 m above the ground

so from the conservation of energy $mg(h_1 - h_2) = \frac{1}{2}mv^2$

$$\Rightarrow v = \sqrt{2g(h_1 - h_2)} = \sqrt{2 \times 10 \times (100 - 20)}$$

$$= \sqrt{1600} = 40 \text{ m/s}$$

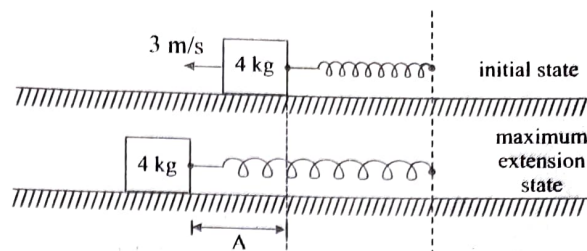
7. (a) Potential energy increases and kinetic energy decreases when the height of the particle increases it is clear from the graph (a).
8. (a) As string does no work on the ball, energy conservation can be applied.

$$\frac{1}{2}mV^2 = mg(L - L \cos \theta)$$

$$V = \sqrt{2g L(1 - \cos \theta)}$$

on putting values $V = \sqrt{10} \text{ m/s}$

9. (c) In the frame (inertial w.r.t. earth) of free end of spring, the initial velocity of block is 3 m/s to left and the spring unstretched.



Applying conservation of energy between initial and maximum extension state.

$$\frac{1}{2}mv^2 = \frac{1}{2}kA^2$$

$$\text{or } A = \sqrt{\frac{m}{k}}v = \sqrt{\frac{4}{1000}} \times 3 = 6 \text{ cm}$$

10. (a) From conservation of energy

$$\text{K.E.} + \text{P.E.} = E$$

$$\text{or } \text{K.E.} = E - \frac{1}{2}kx^2$$

$$\therefore \text{K.E. at } x = -\sqrt{\frac{2E}{k}} \text{ is}$$

$$E - \frac{1}{2}k\left(\frac{2E}{k}\right) = 0$$

$$\therefore \text{The speed of particle at } x = -\sqrt{\frac{2E}{k}} \text{ is zero.}$$

11. (d) As air resistance is negligible, total mechanical energy of the system will remain constant.

Given, $h = 1.5 \text{ m}$, $v = 1 \text{ m/s}$, $m = 10 \text{ kg}$, $g = 10 \text{ ms}^{-2}$

From conservation of mechanical energy.

$$(\text{PE})_i + (\text{KE})_i = (\text{PE})_f + (\text{KE})_f$$

$$\Rightarrow mgh + \frac{1}{2}mv^2 = 0 + (\text{KE})_f$$

$$\Rightarrow (\text{KE})_f = mgh + \frac{1}{2}mv^2$$

$$\Rightarrow (\text{KE})_f = 10 \times 10 \times 1.5 + \frac{1}{2} \times 10 \times (1)^2$$

$$= 150 + 5 = 155 \text{ J}$$

Multiple Correct Answers Type

12. (a, d)

Only the following statements are true from definition of conservative force.

"Its work is zero when the particle moves exactly once around any closed path".

"Its work depends on the end points of the motion, not on the path between".

DPP 8.5

Single Correct Answer Type

1. (d) $P = \vec{F} \cdot \vec{v} = ma \times at = ma^2t$ [as $u = 0$]

$$= m\left(\frac{v_1}{t_1}\right)^2 t = \frac{mv_1^2 t}{t_1^2}$$

[As $a = v_1/t_1$]

$x = OB - \text{natural length of spring} = 2r - r$
 $x = r$

$$\therefore Kr - mg = \frac{mv_B^2}{r}$$

(i)

From conservation of energy,

Total mechanical energy at A = Total mechanical energy at B

Gravitation PE_A + Spring energy PE_A' + Kinetic energy KE_A

= Gravitation PE_B + Spring energy PE_B' + Kinetic energy KE_B

$$mg(BC + CP) + \frac{1}{2}K(0)^2 + 0$$

$$= 0 + \frac{1}{2}kx^2 + \frac{1}{2}mv_B^2$$

$$mg(r + r \cos 60^\circ) = \frac{1}{2}Kr^2 + \frac{1}{2}mv_B^2$$

$$\Rightarrow \frac{3mg}{2} = \frac{1}{2}kr^2 + \frac{1}{2}mv_B^2$$

$$\Rightarrow 3mg = kr^2 + mv_B^2$$

from (i) and (ii)

$$kr^2 - mgr = 3mg - kr^2$$

$$\Rightarrow 2Kr^2 = 4mgr \Rightarrow K = \frac{2mg}{r}$$

$$\Rightarrow K = \frac{2(5)(10)}{(0.2)} \Rightarrow K = 500 \text{ N/m}$$

(ii)

Single Correct Answer Type

2. (d) At highest point $\frac{mv^2}{R} = mg \Rightarrow v = \sqrt{gR}$

3. (d) Kinetic energy given to a sphere at lowest point = potential energy at the height of suspension

$$\Rightarrow \frac{1}{2}mv^2 = mgl$$

$$\therefore v = \sqrt{2gl}$$

4. (d) Since the maximum tension T_B in the string moving in the vertical circle is at the bottom and minimum tension T_T is at the top.

$$\therefore T_B = \frac{mv_B^2}{L} + mg \text{ and } T_T = \frac{mv_T^2}{L} - mg$$

$$\therefore \frac{T_B}{T_T} = \frac{\frac{mv_B^2}{L} + mg}{\frac{mv_T^2}{L} - mg} = \frac{4}{1} \text{ or } \frac{v_B^2 + gL}{v_T^2 - gL} = \frac{4}{1}$$

$$\text{or } v_B^2 + gL = 4v_T^2 - 4gL \text{ but } v_B^2 = v_T^2 + 4gL$$

$$\therefore v_T^2 + 4gL + gL = 4v_T^2 - 4gL \Rightarrow 3v_T^2 = 9gL$$

$$\therefore v_T^2 = 3 \times g \times L = 3 \times 10 \times \frac{10}{3} \text{ or } v_T = 10 \text{ m/sec}$$

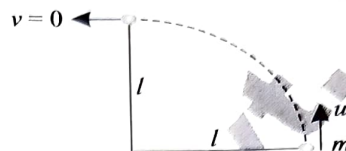
5. (c) Let v be the speed of B at lowermost position, the speed of A at lowermost position is $2v$.

From conservation of energy

$$\frac{1}{2}m(2v)^2 + \frac{1}{2}mv^2 = mg(2\ell) = mg\ell$$

$$\text{Solving we get } v = \sqrt{\frac{6}{5}}g\ell$$

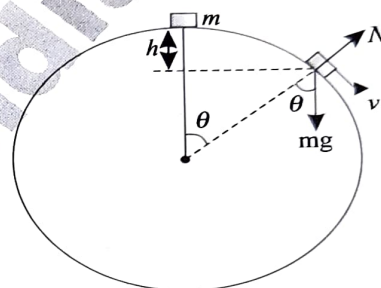
6. (b)



$$\frac{1}{2}mv^2 + mg\ell = \frac{1}{2}mu^2, \text{ for } u_{\min}, v = 0$$

$$u_{\min} = \sqrt{2g\ell}$$

7. (d)



$$h = R - R \cos \theta, v = \sqrt{2gh} = \sqrt{2gR(1 - \cos \theta)}$$

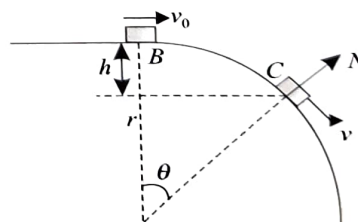
$$mg \cos \theta - N = mv^2/R$$

When it leaves circle : $N = 0$

$$mg \cos \theta = \frac{mv^2}{R} \Rightarrow \cos \theta = \frac{2}{3}$$

$$h = R - R \cos \theta = R/3$$

8. (b)



$$\text{At point C: } mg \cos \theta - N = \frac{mv^2}{r}$$

(when block leaves the surface normal force becomes zero, so

$$\text{putting } N = 0 \Rightarrow g \cos \theta = \frac{v^2}{r}$$

$$v^2 = v_0^2 + 2gh, h = r - r \cos \theta$$

$$gr \cos \theta = (0.5\sqrt{gr})^2 + 2g(r - r \cos \theta)$$

$$\Rightarrow \cos \theta = \frac{1}{4} + 2 - 2 \cos \theta \Rightarrow \cos \theta = \frac{3}{4}$$

9. (c) By law of conservation of mechanical energy

$$(K + U)_A = (K + U)_B$$

$$0 + mgR \cos \alpha = \frac{1}{2}mv_B^2 + mgR \sin \beta$$

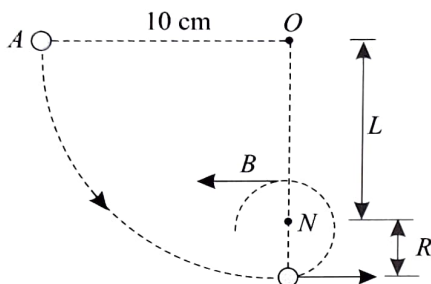
$$v_B^2 = 2gR (\cos \alpha - \sin \beta)$$

At B since particle leaves contact with the surface, hence normal reaction equals zero.

$$mg \sin \beta = \frac{mv_B^2}{R}$$

$$mg \sin \beta = \frac{m}{R}[2gR(\cos \alpha - \sin \beta)]$$

10. (c) Velocity of bob at C should be
- $\sqrt{5gR}$



$$\sqrt{5g(R-L)} = \sqrt{2gR}$$

$$5(R-L) = 2R$$

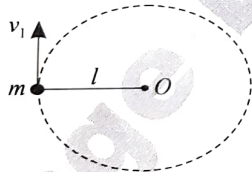
$$3R = 5L$$

$$L = \frac{3}{5}R = \frac{3}{5} \times 15 = 9 \text{ cm}$$

Multiple Correct Answers Type

11. (a, c, d)

For completing circular motion the speed of the particle at top most position should be $v_2 = \sqrt{gl}$, at this position string will slack.



Applying C.O.M.E at 1 and 2

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}mv_2^2 - K_1\right) + mgl = 0$$

$$\frac{1}{2}m(gl) - K_1 = -mgl \Rightarrow K_1 = \frac{3mgl}{2}$$

12. (a, b, c)

Velocity at highest point can be zero, for this velocity at lowest point should be $V_E = \sqrt{4gR}$

$$V_E^2 = V^2 + 2g(R - R \cos 60^\circ)$$

$$4gR = V^2 + gR$$

$$\Rightarrow V = \sqrt{3gR}$$

$$V_F^2 = V_E^2 - 2gR = 4gR - 2gR$$

$$\Rightarrow V_F = \sqrt{2gR}$$

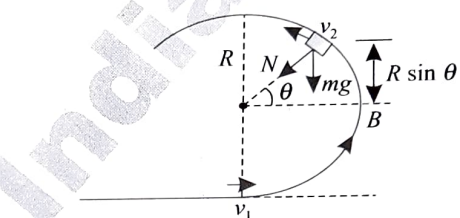
$$N = \frac{mV_F^2}{R} = 2mg$$

$$N_E - mg = \frac{mV_E^2}{R} \Rightarrow N_E = 5mg$$

Comprehension Type

13. (c)
- $v_1 = \frac{2}{\sqrt{5}}\sqrt{5gR} = 2\sqrt{gR}$

$$\text{Now } v_2^2 = v_1^2 - 2g(R + R \sin \theta)$$



$$\Rightarrow v_2^2 = 2gR - 2gR \sin \theta$$

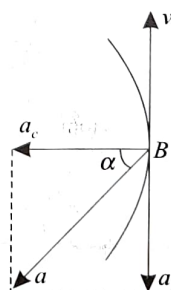
$$N + mg \sin \theta = \frac{mv_2^2}{R}, \text{ put } N = 0$$

$$\Rightarrow mg \sin \theta = m^2 g (1 - \sin \theta)$$

$$\Rightarrow \sin \theta = \frac{2}{3} \Rightarrow \theta = \sin^{-1}\left(\frac{2}{3}\right)$$

14. (a)
- $v^2 = v_1^2 - 2gR \Rightarrow v^2 = 4gR - 2gR$

$$\Rightarrow v = \sqrt{2gR}$$



$$a_c = \frac{v^2}{R} = 2g, \text{ at } g$$

$$\tan \alpha = \frac{a_t}{a_c} = \frac{g}{2g} = \frac{1}{2}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{1}{2}\right)$$

15. (d) Maximum contact force will occur at the lowest point A.